Online appendix for: "Choice Deferral, Indecisiveness and Preference for Flexibility"

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This appendix provides additional material to "Choice Deferral, Indecisiveness and Preference for Flexibility". Section 1 presents our model in a simple, two dimensional case, which also allows us to graphically depict the logic behind our result. In Section 2, this version of the model is used to lay out 3 counterexamples that guarantee the necessity and independence of each of our assumptions; we also show why our result cannot be used to perform comparative statics linking more indecisiveness to stronger desire for flexibility. In Sections 3 and 4, we discuss a strengthening and a weakening of the Cautious Deferral rule and derive conclusions for the identification of the incomplete relation. Finally, Section 5 discusses the relationship between the present framework and the Dominance relation of Kreps (1979).

1. The two-dimensional case

It is well known that under a continuity assumption[1] stronger than our Axiom 2, a complete preference on X satisfies Axiom 1 if and only if it can be represented by a functional $V : X \to \mathbb{R}$ of the form:

$$V(A) = \int_U \max_{a \in A} \mathbb{E}_u(a)\; d\mu(u) \tag{1.1}$$

where U is a set of doubly normalized Bernoulli functions over Z, $\mathbb{E}_u(a)$ is the expectation of u under a, and μ is a countably additive measure over U. When the measure μ has finite support, V assumes a particularly simple form, namely $V(A) = \sum_i^n \alpha_i \max_{a\in A} \mathbb{E}_{u_i}(a)$. In this case, if we let $\Phi(A)$ be the vector $(\max_{a\in A} \mathbb{E}_{u_1}(a), ..., \max_{a\in A} \mathbb{E}_{u_n}(a))$, we can identify every element of X with it's image in $\Phi[X]$, so that the indifference curves induced

[1] For example *L-continuity*, which implies, for a complete preference, Lipschitz continuity of the representing functional (See Dekel, Lipman, Rustichini and Sarver (2007)).

by V can be represented as hyperplanes in $\mathbb{R}^n$ perpendicular to $(\alpha_1, ..., \alpha_n)$.[2] The figure below portrays this situation for the case $n = 2$:

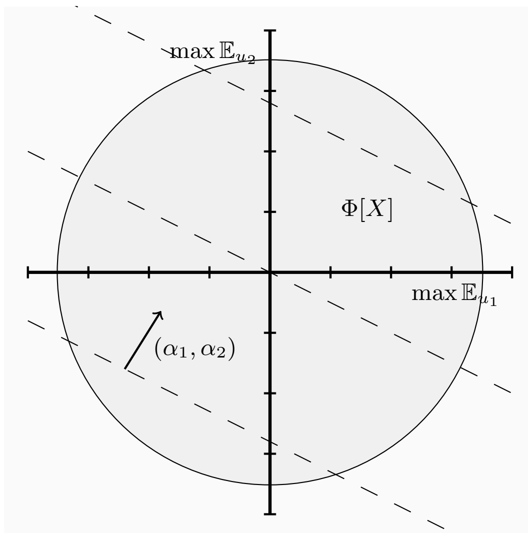


Figure 1.

-A point on the plane is identified with a set $A \in X$ via the correspondence

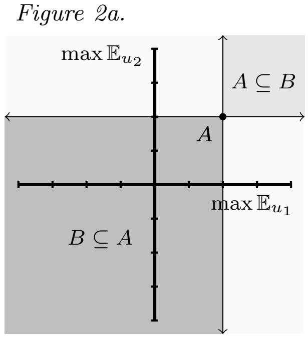
$$\Phi(A) = (\max_A \mathbb{E}_{u_1}, \max_A \mathbb{E}_{u_2}).$$

-The shaded area is the convex set $\Phi[X]$, the image of the domain.

-The dashed lines represent indifference curves for V, each perpendicular to the vector (α_1, α_2).

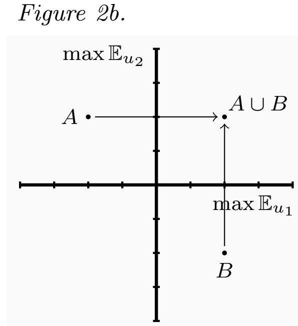
In this representation of the environment, set inclusion can be easily visualized: sets that include A have Φ image in the upper right quadrant with the origin translated to $\Phi(A)$, while sets included in A lay in the lower left quadrant (*Fig.2 a*); moreover, for any two sets A and B, the point representing their union corresponds to the coordinate wise maximum of the vectors $\Phi(A)$ and $\Phi(B)$ (*Fig. 2b*).

Figure 2a.



The light grey area contains the sets including A, the dark grey area those included in A.

Figure 2b.



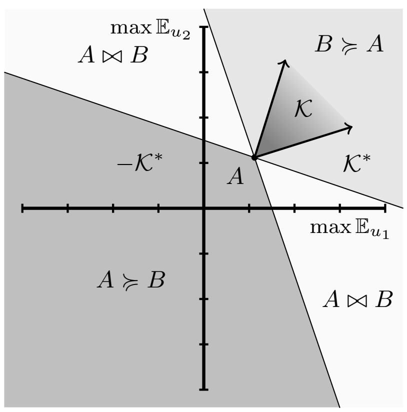
The point corresponding to $\Phi(A \cup B)$ coincides with $\max\{\Phi(A), \Phi(B)\}$.

Also the incomplete preorder $\succcurlyeq$ can be visualized in the above graph, as long as some additional conditions are satisfied. In fact, as is shown in the appendix of

[2]In fact it can be proved that whenever μ has support of finite cardinality n, given an appropriate normalization for U, the image $\Phi[X]$ is a convex subset of $\mathbb{R}^n$ containing the origin, while $\text{Span}(\Phi[X]) = \{\lambda(\Phi[X] - \Phi[X]) \mid \lambda > 0\}$ is equal to $\mathbb{R}^n$.

the main text, under our assumptions, $\succcurlyeq$ is the intersection of a collection of complete preorders $\{\succcurlyeq_i\}_{i\in I}$ satisfying Continuity and Independence. If in addition each such preorder has an integral representation such as V above, and if the support of the measures corresponding to such V_i's is included in $\{u_1, u_2\}$, we can identify every $\succcurlyeq_i$ with the pair weights (α_1^i, α_2^i) it assigns to each Bernoulli utility. In this case a set B is preferred to A for every $\succcurlyeq_i$ (and thus for $\succcurlyeq$) only if it is contained in the intersection of all half-spaces corresponding to the upper contour sets of A under each $\succcurlyeq_i$, namely the translate of $\mathcal{K}^*$, the dual cone of $\mathcal{K} = \{(\alpha_1^i, \alpha_2^i) \mid V_i = \sum_j \alpha_j^i \max \mathbb{E}_{u_j} \text{ for some } i \in I\}$. At the same time one can see that any proper completion of $\succcurlyeq$ will correspond to a vector (α_1, α_2) contained in $\mathcal{K}$:

Figure 3a.

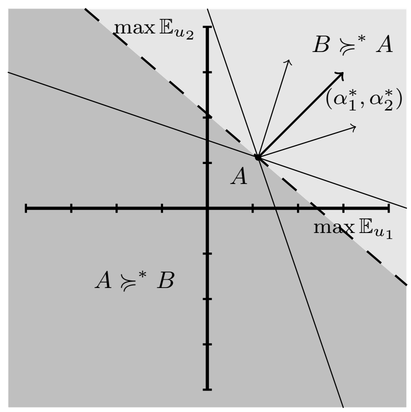


-The shaded cone is the one generated by the vectors in $\mathcal{K}$ translated to A.

-The dual $\mathcal{K}^*$, in grey, contains all sets preferred to A under $\succcurlyeq$.

-The negative of the dual, highlighted in dark grey, contains all sets B that are $\succcurlyeq$-dominated by A.

Figure 3b.



-The thick dashed line is the indifference curve of a possible completion $\succcurlyeq^*$ of $\succcurlyeq$ going through A.

-The normal to such indifference curve identifies the functional $V^* = \sum_i \alpha_i^* \max \mathbb{E}_{u_i}$ representing the completion.

We provide below an alternative proof of Theorem 1, which works specifically in the finite dimensional case and which will be useful when we introduce our counterexamples.

1.1. Proof Sketch

There are two distinct cases to consider:

Case I:
Assume there are sets $A \bowtie B$ such that $\max_B \mathbb{E}_{u_1} < \max_A \mathbb{E}_{u_1}$ and $\max_B \mathbb{E}_{u_2} >$

$\max_A \mathbb{E}_{u_2}$. In this case, the functional V^* of a completion satisfying Cautious Deferral will necessarily need to satisfy

$$V^*(A \cup B) - V^*(B) = \alpha_1^*(\max_A \mathbb{E}_{u_1} - \max_B \mathbb{E}_{u_1}) \geq 0 \Longrightarrow \alpha_1^* \geq 0$$

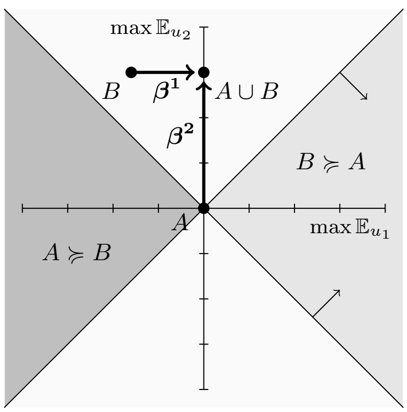
and

$$V^*(A \cup B) - V^*(A) = \alpha_2^*(\max_B \mathbb{E}_{u_2} - \max_A \mathbb{E}_{u_2}) \geq 0 \Longrightarrow \alpha_2^* \geq 0$$

Thus the measure over $\{u_1, u_2\}$ that identifies V^* is positive, which means, as shown in DLR, that the preference $\succcurlyeq^*$ is monotone.[3]

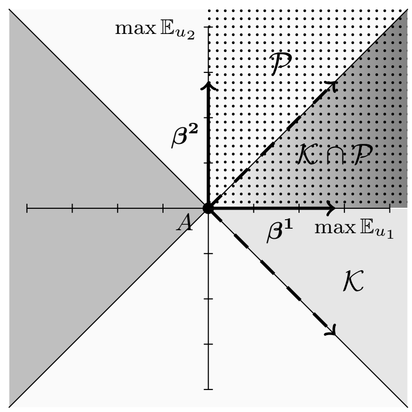
The argument is illustrated in Figure 4 below, where we are assuming for simplicity that $\Phi(A) = (0, 0)$:

Figure 4a.



Moving from B in direction $\boldsymbol{\beta^1}$, or from A in direction $\boldsymbol{\beta^2}$, must be an improvement for any completion $\succcurlyeq^*$ satisfying Cautious Deferral.

Figure 4b.



-The set of directions spanned by $\{\boldsymbol{\beta^1}, \boldsymbol{\beta^2}\}$ coincides with the positive cone $\mathcal{P}$, patterned in dots.

-The thick dashed arrows delimit the cone $\mathcal{K}$ of possible directions in which a completion of $\succcurlyeq$ must lie.

-A direction in the shaded area $\mathcal{K} \cap \mathcal{P}$ satisfies both restrictions, and thus identifies an admissible completion.

Here, Cautious Deferral restricts the directions that the completion $\succcurlyeq^*$ can take, forcing it to lie in the cone at the intersection between $\mathcal{K}$ and the positive cone $\mathcal{P}$.

[3] That the representation of $\succcurlyeq^*$ should be of the form $\alpha_1^* \max \mathbb{E}_{u_1} + \alpha_2^* \max \mathbb{E}_{u_2}$ can be deduced as a consequence of an aggregation theorem a là Harsanyi. Simply put, if it where not the case, there should be A and B such that $\max_A \mathbb{E}_{u_i} = \max_B \mathbb{E}_{u_i}$ for $i = 1, 2$ but $A \succ^* B$. But given the restrictions we assumed on the representation of $\succcurlyeq$, the previous equalities imply $A \sim B$, which would necessarily lead to $A \sim^* B$, a contradiction.

Case II:
Suppose Case I does not hold. Then if $A \bowtie B$, either $\max_A \mathbb{E}_{u_i} \leq \max_B \mathbb{E}_{u_i}$ or $\max_B \mathbb{E}_{u_i} \leq \max_A \mathbb{E}_{u_i}$ for $i = 1, 2$. Notice that, since for every set A, the set $\dot{A} = \bigcap_i \{b \in X \mid \mathbb{E}_{u_i}(b) \leq \max_{a \in A} \mathbb{E}_{u_i}(a)\}$ must have the same image under Φ as A, this implies that for every incomparable pair (A, B) the corresponding incomparable pair $(\dot{A}, \dot{B})$ satisfies either $\dot{A} \subset \dot{B}$ or $\dot{B} \subset \dot{A}$.

Now assume that two incomparable sets (A, B) are indifferent under the completion $\succcurlyeq^*$. The lemma below shows that in our restricted environment, and except for trivial situations, every completion admits one such pair:

Lemma 1. *Let $\succcurlyeq^*$ and each $\succcurlyeq_i$ in the collection $\{\succcurlyeq_i\}_{i \in \mathcal{I}}$ such that*

$$\succcurlyeq = \bigcap_{i \in \mathcal{I}} \succcurlyeq_i$$

have a representation of the form (1.1), and let all corresponding measures have support in some finite set of Bernoulli utilities. Then if $\bowtie$ and $\succ$ are nonempty, there are $A, B \in X$ such that $A \bowtie B$ and $A \sim^* B$.

Proof: Suppose the statement is false. Then for all $A \in X$, we have that

$$\{B \in X \mid B \sim^* A\} \subseteq \{B \in X \mid B \sim A\} = \bigcap_{i \in \mathcal{I}} \{B \in X \mid B \sim_i A\}. \qquad (1.2)$$

Let $\boldsymbol{\alpha^*}$ and $\boldsymbol{\alpha^i}$ be the vectors in $\mathbb{R}^n$ corresponding to the representations of $\succcurlyeq^*$ and $\succcurlyeq_i$ respectively. Then, letting A be such that $\Phi(A) = \mathbf{0}$, line (1.2) is equivalent to

$$\{\Phi(B) \in \Phi[X] \mid \boldsymbol{\alpha^*} \cdot \Phi(B) = 0\} \subseteq \bigcap_{i \in \mathcal{I}} \{\Phi(B) \in \Phi[X] \mid \boldsymbol{\alpha_i} \cdot \Phi(B) = 0\}.$$

Because $\succ$ is nonempty, $\boldsymbol{\alpha^*}$ must be a non-zero vector. Thus, as long as $\boldsymbol{\alpha^i}$ is different from zero, the sets $\{\Phi(B) \in \Phi[X] \mid \boldsymbol{\alpha^*} \cdot \Phi(B) = 0\}$ and $\{\Phi(B) \in \Phi[X] \mid \boldsymbol{\alpha_i} \cdot \Phi(B) = 0\}$ are the intersection between $\Phi[X]$ and an $n-1$ dimensional subspace of $\mathbb{R}^n$. Since $\Phi[X]$ spans $\mathbb{R}^n$, basic dimensionality arguments imply that for every nontrivial $\succcurlyeq_i$ we must have:

$$\{\Phi(B) \in \Phi[X] \mid \boldsymbol{\alpha^*} \cdot \Phi(B) = 0\} = \{\Phi(B) \in \Phi[X] \mid \boldsymbol{\alpha_i} \cdot \Phi(B) = 0\}.$$

This means that for each $i \in \mathcal{I}$ there is a $\lambda_i \in \mathbb{R}$ such that $\boldsymbol{\alpha^i} = \lambda_i \boldsymbol{\alpha^*}$. Non-emptiness of $\succ$ implies that some λ_i must be different from zero. If each

λ_i is non-negative all relations $\succcurlyeq_i$ agree, so $\bowtie$ must be empty. Similarly if each λ_i is non-positive. Finally, if some λ are strictly positive and some are strictly negative, the relation $\succcurlyeq$ is the intersection of the preferences $\succcurlyeq_+$ and $\succcurlyeq_-$ induced by $\boldsymbol{\alpha^*}$ and $-\boldsymbol{\alpha^*}$. In this case, since $A \succ_+ B$ implies $B \succ_- A$, we cannot have nonempty $\succ$. ■

W.l.o.g. we have $\dot{A} \subset \dot{B}$. Because under closed continuity incomparability is open, there is an open neighborhood $\mathcal{O}_B$ of B such that $A \bowtie C$ whenever $C \in \mathcal{O}_B$. Because $\Phi[X]$ spans $\mathbb{R}^2$ this implies that for every direction $\boldsymbol{\beta}$, there is a small enough ϵ such that $\Phi(B) + \epsilon\boldsymbol{\beta}$ is included in $\Phi[\mathcal{O}_B]$.

In particular, setting $\boldsymbol{\beta} = -(\alpha_1^*, \alpha_2^*)$ we see that in any open neighborhood of B we can find a set B_1 that is incomparable to A and such that $B \succ^* B_1$. Continuity of set inclusion in the Hausdorff topology implies we can find a neighborhood small enough such that $\dot{A} \subset \dot{B}$ implies $\dot{A} \subset \dot{B}_1$. But then $\dot{A} \bowtie \dot{B}_1$ but $\dot{A} \succ^* \dot{A} \cup \dot{B}_1 = \dot{B}_1$, contradicting Cautious Deferral. Figure 5 below illustrates the case:

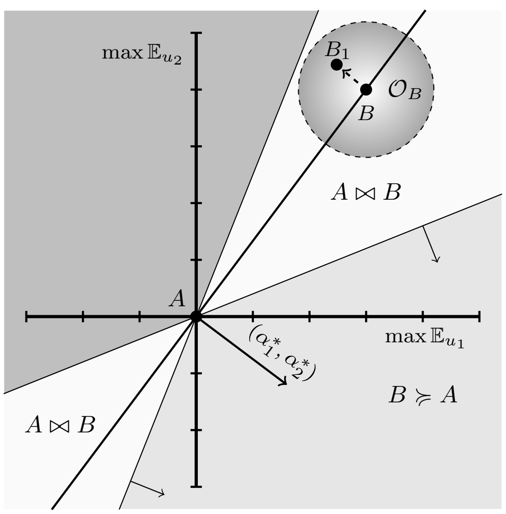


Figure 5.

-The indifference curve induced by $\succcurlyeq^*$, must contain some set $B \bowtie A$.

-The shaded open ball $\mathcal{O}_B$ contains only sets incomparable to A.

-The set B_1 is strictly worse than A under $\succcurlyeq^*$. At the same time $\dot{A} \subset \dot{B}_1$.

As is clear from the picture, in this situation any proper completion is inadmissible.

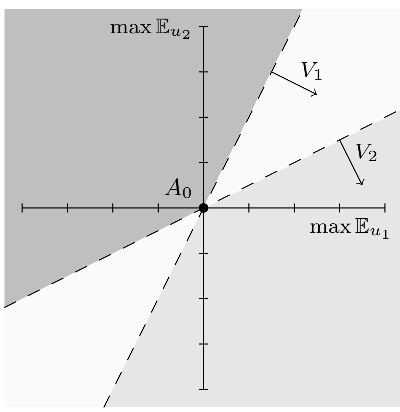
2. Counterexamples

Here we use the simple environment sketched above to provide counterexamples showing that Axioms 1 and 2 are necessary for our results. We do not provide a counterexample for Axiom 3 since in this case necessity is quite immediate. We do instead show that if one relaxes the requirement that the completion $\succsim^*$ be proper, our result does not hold.

2.1. Counterexample 1: Continuity

Define a relation $\rhd$ in the following way: $A \rhd B$ if and only if $V_1(A) > V_1(B)$ and $V_2(A) > V_2(B)$, where $V_1(A) = 2\max_A \mathbb{E}_{u_1} - \max_A \mathbb{E}_{u_2}$ and $V_2(A) = \max_A \mathbb{E}_{u_1} - 2\max_A \mathbb{E}_{u_2}$. Because V_1 and V_2 are real valued functions of the form 1.1, $\rhd$ is transitive and satisfies Axiom 2. But for each i, the relation "A preferred to B iff $V_i(A) > V_i(B)$ " is an open subset of $X \times X$. Hence, also $\rhd$ is open, which violates Axiom 1. Figure 6a below shows the upper and lower contour sets of $\rhd$ at the set $A_0 \in \Phi^{-1}(\mathbf{0})$.

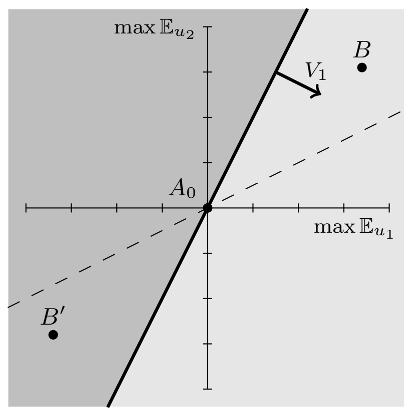
Figure 6a.



-The two dashed lines represent indifference curves corresponding to V_1 and V_2.

-The light grey and dark grey cones, corresponding to upper and lower contour sets of $\rhd$, are open.

Figure 6b.



-Sets incomparable to A_0 that include A_0 are like B, so $B \trianglerighteq_1 A$.

-Sets incomparable to A_0 and included in A_0 are like B', so $A \trianglerighteq_1 B'$.

Now consider the relation $\trianglerighteq_1$ given by $A \trianglerighteq_1 B$ if and only if $V_1(A) \geq V_1(B)$. This relation is open (and closed) continuous, complete, transitive, and satisfies Axiom 2. Moreover if $A \rhd B$ then necessarily $A \rhd_1 B$, so $\trianglerighteq_1$ is a proper completion of $\rhd$. Nevertheless it is non-monotonic. It remains to show that it satisfies Cautious Deferral. To do so, take any set B such that $A_0 \subset B$. It follows that $\max_B \mathbb{E}_{u_i} \geq 0$ for $i = 1, 2$. If $B \bowtie A_0$, it must be that either $V_i(A_0) = V_i(B)$ for some $i = 1, 2$ or $V_i(A_0) > V_i(B)$ and $V_j(A_0) < V_j(B)$. Now let $\Phi(B) = (x_1, x_2)$ and remember that $V_i(A_0) = 0$.

If $V_2(B) = 0$ then $x_1 = 2x_2$ which, with non-negativity of the x_i's means that $2x_1 - x_2 = 3x_2 \geq 0$, which implies $B \trianglerighteq_1 A$. If $V_1(B) = 0$ then obviously $B \trianglerighteq_1 A$. Of the remaining to cases, $V_2(A_0) > V_2(B)$ and $V_1(A_0) < V_1(B)$ already means $B \trianglerighteq_1 A_0$, while $V_1(A_0) > V_1(B)$ and $V_2(A_0) < V_2(B)$ cannot hold, since the latter inequality implies $x_1 > 2x_2$ and forcefully, by non-negativity of x_2, also $V_1(B) \geq 0$. So $B \bowtie A_0$ and $A_0 \subseteq B$ imply $B \trianglerighteq_1 A_0$ as required. A symmetric argument applies to sets that are included in A_0, and the whole discussion can be applied to any point in $\Phi[X]$.

As can be seen in Figure 6b, out of all sets that are incomparable to A_0, all those that might contain A_0 are $\trianglerighteq_1$ preferred to A_0, and all those that might be contained in A_0 are $\trianglerighteq_1$ worse than A_0. This can happen here because the proper completion of an open continuous relation can correspond to one of the indifference curves that form the extreme rays of the upper and lower contour sets. Thus the contradiction that arises in Case II of the proof of Section 1 has no bite.

2.2. Counterexample 2: Proper Completion

We will follow a logic that is very close to the one presented for the first counterexample. Let $\trianglerighteq$ be the relation given by $A \trianglerighteq B$ if and only if $V_i(A) \geq V_i(B)$ for $i = 1, 2$, where the V_i's are those defined in Counterexample 1. Now $\trianglerighteq$ is closed continuous and thus it satisfies all our assumptions. At this point, consider the relation $\trianglerighteq_1$ induced by V_1. As we showed above, it satisfies Axioms 1-3 (because we are reducing the number of incomparable sets, Cautious Deferral must be satisfied also here). But $\trianglerighteq_1$ is not a proper completion of $\trianglerighteq$. In fact, consider set C such that $\Phi(C) = (-0.5, -1)$. We have $V_2(C) > 0$ since $V_2(C) = -0.5 + 2$. On the other hand $V_1(B) = -1 + 1 = 0$ so $A_0 \sim_1 B$. But since this implies $C \triangleright A_0$, we have found a point in $\triangleright$ that is not in $\triangleright_1$. Nevertheless V_1 represents a complete extension of $\trianglerighteq$ since obviously $\trianglerighteq \subseteq \trianglerighteq_1$.

2.3. Counterexample 3: Independence

There are many ways in which the theorem might not go through if we weaken Independence. Here we concentrate on one of them, mainly the fact that when Independence is lost, local properties of the preferences need not extend globally. In particular, one reason we obtain such a strong result is that, under Independence, if there exists at least two menus which are incomparable, then for almost every menu, we can find a nearby menu that is incomparable to it (this is not necessarily true if the menu is at the boundary of the domain, hence the "almost" qualifier).

We will thus provide an example in which this is not true anymore, and show that in this case a completion that satisfies Cautious Deferral can be be non-monotonic. To keep the intuition simple, we will relax Independence to Indifference to Randomization (that is, maintain the convexity property behind Independence but relax linearity). Since the exact domain of the preferences is now important (so that we need to look at all points in the space, not just at a representative one), it will be useful to identify the underlying utilities.

We assume a space with three prizes, so $\triangle := \triangle(\{z_1, z_2, z_3\})$. Our incomplete relation $\succcurlyeq$ is given by the intersection of two relations represented by $V_1(\max_{(\cdot)} \mathbb{E}_u, \max_{(\cdot)} \mathbb{E}_v)$ and $V_2(\max_{(\cdot)} \mathbb{E}_u, \max_{(\cdot)} \mathbb{E}_v)$, where u and v are the EU functionals induced by Bernoulli utilities $u = \{\frac{1}{\sqrt{3}}, -\frac{1}{2\sqrt{3}}, -\frac{1}{2\sqrt{3}}\}$ and $v = \{-\frac{1}{2\sqrt{3}}, -\frac{1}{2\sqrt{3}}, \frac{1}{\sqrt{3}}\}$.

It is immediate to see that $(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}) = \Phi(\{(1,0,0),(0,0,1)\}) \geq \Phi(A)$ for all $A \in X$ and that

$$(-\frac{1}{2\sqrt{3}}, -\frac{1}{2\sqrt{3}}) = \Phi(\{(0,1,0)\}) \leq \Phi(A)$$

for all $A \in X$. Finally

$$(\frac{1}{\sqrt{3}}, -\frac{1}{2\sqrt{3}}) = \Phi(\{(1,0,0)\}) = (\max_{A \in X} \max_{a \in A} \mathbb{E}_u, \min_{B \in X} \max_{b \in B} \mathbb{E}_v)$$

and

$$(-\frac{1}{2\sqrt{3}}, \frac{1}{\sqrt{3}}) = \Phi(\{(0,0,1)\}) = (\min_{A \in X} \max_{a \in A} \mathbb{E}_u, \max_{B \in X} \max_{b \in B} \mathbb{E}_v)$$

Thus as can be seen in Figure 7, the set $\Phi[X]$ will be a square of length $\frac{\sqrt{3}}{2}$, with the bottom left corner at $(-\frac{1}{2\sqrt{3}}, -\frac{1}{2\sqrt{3}})$.

We assume that V_1 is defined by

$$V_1(A) = -[(\max_A \mathbb{E}_u - \frac{\sqrt{3}}{4})^2 + (\max_A \mathbb{E}_v - \frac{\sqrt{3}}{4})^2]$$

Here the utility is the opposite of the euclidean distance (in $\Phi_{u,v}[\triangle]$)

from $(\frac{\sqrt{3}}{4}, \frac{\sqrt{3}}{4})$.

We will take V_2 to be

$$V_2 = \begin{cases} V_1 & \text{if } V_1(A) > V_1(A^*), \\ V_1 + (V_1 - V_1(A^*))e^{\max \mathbb{E}_u + \max \mathbb{E}_v} & \text{if } V_1(A) \leqslant V_1(A^*). \end{cases}$$

As can be seen V_2 is taken to be identical to V_1, except for a region below a given indifference curve corresponding to some menu A^*. Below such level, V_2 is distorted by $(V_1(A) - V_1(A^*))e^{\max_A U + \max_A \mathbb{E}_v}$. This distortion is negative whenever $V_1(A) < V_1(A^*)$, continuously converging to zero as $V_1(A) \to V_1(A^*)$, and has the easily verified property that, whenever V_1 is monotone in $(\mathbb{E}_u, \mathbb{E}_v)$, so is $V_1 + (V_1 - V_1(A^*))e^{\max \mathbb{E}_u + \max \mathbb{E}_v}$. Letting $\frac{5\sqrt{6}}{12} > V_1(A^*) > \frac{\sqrt{78}}{12}$ ensures that the indifference curves of V_2 will differ from those of V_1 only in the lower left corner of the previous graph (the indifference curve that touches the southern and western boundary of the domain corresponds to $V_1(A) = \frac{5\sqrt{3}}{12}$). Figure 7 shows the indifference curves of both functions, highlighting in dark grey the area where they differ:

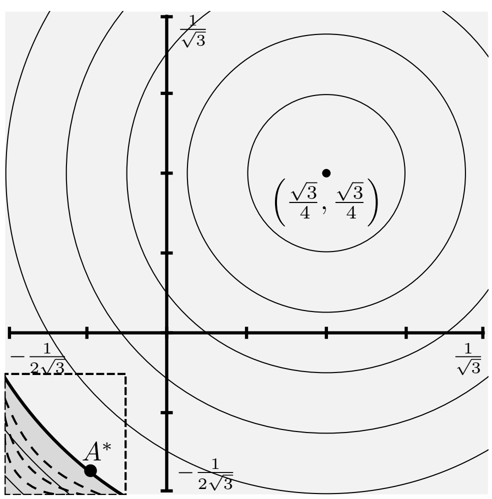


Figure 7.

-Any completion will be non-monotonic in the upper right corner of $\Phi[X]$.

-The Φ image of the union of any two incomparable sets will lie in the dotted square in the lower left corner.

Here the thicker indifference curve corresponds to $V_1(A^*)$, the red area corresponds to the zone in which there will be indecisiveness and the dashed lines are the indifference curves of V_2. As can be noticed, any set corresponding to $\max\{\Phi(A), \Phi(B)\}$ for two sets A, B that are incomparable is in an area where both functions V_1 and V_2 are increasing with respect to $(\mathbb{E}_u, \mathbb{E}_v)$, and thus monotonic. This means that the Cautious Deferral Axiom does not impose any additional restriction on the completion. Thus, it will be sufficient to set, for example, $G = 0.5V_1 + 0.5V_2$ to obtain a completion satisfying all our requirements. Notice nevertheless that G coincides with V_1 outside of the red area and it is thus non monotonic in some parts of the domain (e.g. in the upper right hand corner).

2.4. On Comparative Statics

In light of our result, a natural question that arises is whether a higher level of indecisiveness will lead to a stronger desire for flexibility. Here we show that the Cautious Deferral axiom is in fact too weak to allow us to perform this kind of comparative statics.

To fix the language, we will follow DLR (Section 3.1, p. 909) and say that $\succcurlyeq_1^*$ *strictly desires more flexibility* than $\succcurlyeq_2^*$ if (i) $A \subset B$ and $B \succ_2^* A$ implies $B \succ_1^* A$; (ii) there is some $A' \subset B'$ such that $B' \succ_1^* A'$ and $A' \succcurlyeq_2^* B'$.

We now show in the following counterexample that under the assumptions of our theorem, it is possible that $\succcurlyeq_1^*$ strictly desires more flexibility than $\succcurlyeq_2^*$ even if $\succcurlyeq_2^*$ is more incomplete than $\succcurlyeq_1^*$. To see this point, let

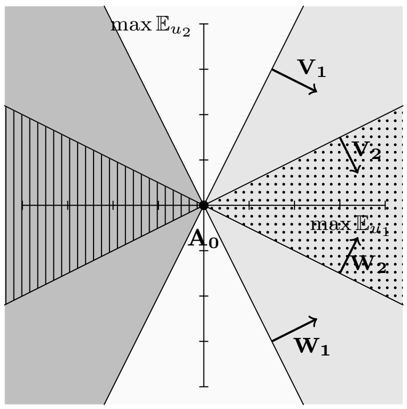
$$V_1(A) = 2 \max_A \mathbb{E}_{u_1} - \max_A \mathbb{E}_{u_2}, \quad W_1(A) = 2 \max_A \mathbb{E}_{u_1} + \max_A \mathbb{E}_{u_2}$$

and

$$V_2(A) = \max_A \mathbb{E}_{u_1} - 2 \max_A \mathbb{E}_{u_2}, \quad W_2(A) = \max_A \mathbb{E}_{u_1} + 2 \max_A \mathbb{E}_{u_2}$$

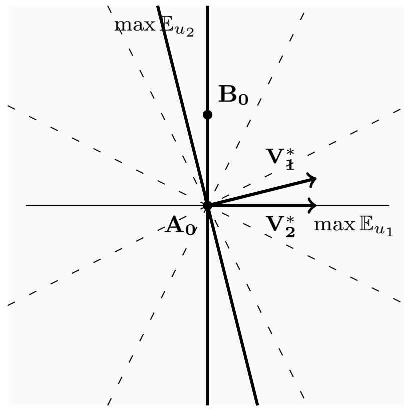
and consider two decision makers, DM1 and DM2, with psychological preferences $\succcurlyeq_1$ and $\succcurlyeq_2$ given respectively by $A \succcurlyeq_1 B$ iff $V_1(A) \geq V_1(B)$ and $W_1(A) \geq W_1(B)$, and by $A \succcurlyeq_2 B$ iff $V_2(A) \geq V_2(B)$ and $W_2(A) \geq W_2(B)$.

Figure 8a.



Upper and lower contour sets at A_0 for $\succcurlyeq_1$ and $\succcurlyeq_2$, shown in light/dark grey and dotted/vertical line patterns.

Figure 8b.



The two thick dark lines represent the indifference curves through A_0 induced by V_1^* and V_2^*.

Figure 8a shows the upper and lower contour sets induced by both relations at menu A_0 satisfying $\Phi(A_0) = (0, 0)$. The upper contour set at A_0 under $\succcurlyeq_1$, highlighted in light grey, coincides with $\{B \in X \mid V_1(B) \geq V_1(A_0) \text{ and } W_1(B) \geq W_1(A_0)\}$. The lower contour set is highlighted in dark grey. Upper and lower contour sets at A_0 for $\succcurlyeq_2$ are highlighted with dotted and vertical line patterns respectively.

As can be seen from the picture, at A_0, the upper contour set induced by $\succcurlyeq_2$ is included in the one induced by $\succcurlyeq_1$. Since, by Independence, these sets are in a certain sense "translation invariant"[4], this implies that DM2 is more indecisive than DM1. An other way to say this is that at every set A, the collection of sets incomparable to A under $\succcurlyeq_2$ includes the collection of sets incomparable to A under $\succcurlyeq_1$.

Now let $V_1^* = 2 \max \mathbb{E}_{u_1} + \frac{1}{2} \max \mathbb{E}_{u_2}$ and $V_2^* = \max \mathbb{E}_{u_1}$. Since $V_1^* = \frac{3}{4} W_1 + \frac{1}{4} V_1$ and $V_2^* = \frac{1}{2} W_2 + \frac{1}{2} V_2$, these functionals induce two relations $\succcurlyeq_1^*$ and $\succcurlyeq_2^*$ that are (continuous and affine) proper completions of $\succcurlyeq_1$ and $\succcurlyeq_2$, respectively. Moreover, since they are both monotone, the preference structures $(\succcurlyeq_1, \succcurlyeq_1^*)$ and $(\succcurlyeq_2, \succcurlyeq_2^*)$ also satisfy Cautious Deferral.

Finally, notice that since $B \succ_2^* A$ implies $\max_B \mathbb{E}_{u_1} > \max_A \mathbb{E}_{u_1}$ by monotonicity of the max operator, $A \subset B$ and $B \succ_2^* A$ implies $B \succ_1^* A$. On the other hand, consider the pair (A_0, B_0) where $\Phi(B_0) = (0, 1)$ and

[4]To clarify, consider $\mathcal{K}_i = \{\lambda(\Phi(B) - \Phi(A)) \parallel \lambda \geq 0 \text{ and } B \succcurlyeq_i A\}$, the dominance cone induced by $\succcurlyeq_i$. Then affinity of the V_i and W_i functionals, which is guaranteed by Independence, ensures that the Φ image of the upper contour set of any A under $\succcurlyeq_i$ coincides with $\Phi(A) + \mathcal{K}_i \cap \Phi[X]$.

$A_0 \in \Phi^{-1}((0,0))$ as previously. Recall that for any set A we can define a set

$$\dot{A} = \{a \in \triangle \mid \mathbb{E}_{u_i}(a) \leq \max_A \mathbb{E}_{u_i} \text{ for } i = 1,2\}$$

which will be indifferent to A under both relations. These sets have the important property that $\Phi(\dot{A}) \leq \Phi(\dot{B})$ if and only if $\dot{A} \subseteq \dot{B}$. Hence the sets $\dot{A}_0$ and $\dot{B}_0$ will satisfy $\dot{A}_0 \subset \dot{B}_0$ and $\dot{B}_0 \succ_1^* \dot{A}_0$, but $\dot{A}_0 \sim_2^* \dot{B}_0$. Thus, while DM2 is more indecisive than DM1, here DM1 strictly desires more flexibility than DM2. Thus, more indecisiveness does not necessarily correlate with a stronger preference for flexibility in our model. An illustration of the two different completions is given in figure 8b.

3. Strong Cautious Deferral

The Cautious Deferral rule posits a very weak link between indecisiveness and choice deferral, in two precise senses. On the one side one might not strictly prefer to postpone when unable to compare two menus, as $A \cup B \sim A$ and $A \hat{\bowtie} B$ is allowed. On the other we might have pairs A, B at which $\succcurlyeq$ exhibits strict preference for flexibility even though $A \hat{\succcurlyeq} B$. In this section we study the consequences of strengthening our definition in both directions, which leads to the following:

Definition: A complete and rational preference $\succcurlyeq$ on X is a *Strong Cautious Deferral completion* if there exists a rational preference $\hat{\succcurlyeq}$ on X such that:

(*C1*) $\succcurlyeq$ is a proper completion of $\hat{\succcurlyeq}$.

(*D2*) $A \hat{\bowtie} B$ if and only if $A \cup B \succ A, B$.

As condition *D2* is obviously stronger than the Cautious Deferral rule, by Proposition 1 in the paper only monotonic preferences $\succcurlyeq$ can be Strong Cautious Deferral completions of non-trivially incomplete relations. We present the additional implications of this definition below. Before doing so,we need to introduce some notation. Let $\mathcal{U}^*$ be the set of doubly normalized Bernoulli utilities over Z, namely the set of functions $u \in \mathbb{R}^Z$ such that $\sum_{z \in Z} u(z) = 0$ and $\sum_{z \in Z} u(z)^2 = 1$, endowed with the topology inherited from the Euclidean space $\mathbb{R}^Z$. Let $\mathbb{E}_u(a) = \sum_{z \in Z} a(z)u(z)$. Then:

Proposition 1. *Let $\succcurlyeq$ be a complete, and rational preference on X. Then the following are equivalent:*

1) $\succcurlyeq$ *is the Strong Cautious Deferral completion of some relation* $\hat{\succcurlyeq}$ *that is incomplete.*

2) $\succcurlyeq$ *exhibits strict preference for flexibility at some pair* A, B*.*

3) There is a nonempty, non-singleton closed set $\mathcal{U} \subseteq \mathcal{U}^*$ *such that*

$$A \,\hat{\succcurlyeq}\, B \Longleftrightarrow \max_{a \in A} \mathbb{E}_u(a) \geq \max_{b \in B} \mathbb{E}_u(b) \text{ for all } u \in \mathcal{U} \qquad (3.1)$$

The above $\mathcal{U}$ *is unique in the sense that if another nonempty, closed set* $\mathcal{V} \subseteq \mathcal{U}^*$ *satisfies 3.1 then* $\mathcal{V} = \mathcal{U}$*. Notice this implies that* $\hat{\succcurlyeq}$ *is uniquely identified.*

The proof of this proposition is available from the authors upon request. The intuition is that under Strong Cautious Deferral, as long as there is some underlying incompleteness a) $\succcurlyeq$ must be monotone and b) the relation $\hat{\succcurlyeq}$ must coincide with the Krepsian dominance relation (see Section 5). Under Independence the latter has a representation that can be identified using the support of the measure associated to the DLR representation. We note that Danan (2003b) had already provided a result along these lines.

4. Simple Cautious Deferral

Consider the following weakening of our definiton of Cautious Deferral completion:

Definition: A complete and rational preference $\succcurlyeq$ on X is a *Simple Cautious Deferral completion* if there exists a rational preference $\hat{\succcurlyeq}$ on X such that:

(*C1*) $\succcurlyeq$ is a proper completion of $\hat{\succcurlyeq}$.

(*C2'*) $\{p\} \hat{\bowtie} A$ implies $A \cup \{p\} \succcurlyeq \{p\}$ for all $p \in \triangle$ and $A \in X$

The new rule in *C2'*, simply requires that the agent does nto choose to commit to a single option $\{p\}$ when he is indecisive between p and A and is allowed to postpone by choosing $A \cup \{p\}$. Under some additional restrictions on the complete preference $\succcurlyeq$, we can show that a similar result to Proposition 1 in the paper also holds with this weaker definition. First, we can prove the following claim:

Claim 1. Let $\succcurlyeq$ be a Simple Cautious Deferral completion on X. Then as long as $\{p\} \succ \{q\}$ for some $p, q \in \triangle$, either $D \succcurlyeq \{c\}$ for all $c \in D$ and

$D \in X$, or $\succcurlyeq$ and $\hat{\succcurlyeq}$ coincide over singleton sets.

Proof of Claim 1. In the following we denote, abusing notation, singletons $\{p\}$ with p. Let $\succcurlyeq$ be a Simple Cautious Deferral completion on X such that $p \succ q$ for some $p, q \in \triangle$. If $\succcurlyeq$ and $\hat{\succcurlyeq}$ do not coincide on singletons, it must be that $a \hat{\bowtie} b$ for some $a, b \in X$. Suppose by contradiction that $c \succ D$ for some $c \subset D$. As $\succcurlyeq$ is non-trivial on singletons, we can assume w.l.o.g. that $a \succ b$. Also, as always we can assume D is closed and convex. Now we can replicate the construction of the Proposition 1 proof. We obtain

$$e = \frac{1}{2}c + \frac{1}{2}a$$

$$f = \frac{1}{2}c + \frac{1}{2}b$$

$$G = \frac{1}{2}D + \frac{1}{2}b$$

$$H = \frac{1}{2}c + \frac{1}{2}\overline{co}(\{a, b\}).$$

Replicatng the steps of the proof, we obtain either a set J such that $f \succ J$ but $f \subset J$ and $f \hat{\bowtie} J$, or in the alternative case a singleton $k = \alpha e + (1-\alpha)f$ contained in a set $I = \alpha H + (1-\alpha)G$ for some α, such that $k \hat{\bowtie} I$ and $k \succ I$, violating in either situation the Simple Cautious Deferral rule. ■

As the following shows, when $\succcurlyeq$ has a finite subjective state representation, we can then prove a modified version of Proposition 1 for the weaker Simple Cautious Deferral:

Claim 2. Let $\succcurlyeq$ be a Simple Cautious Deferral completion on X that is non-trivial on singletons and has a finite subjective state representation. Then if $\{p\}\hat{\bowtie}\{q\}$ for some $p, q \in \triangle$, the relation $\succcurlyeq$ must be monotonic.

Proof that Claim 1 $\rightarrow$ Claim 2. As we know, when $\succcurlyeq$ has a fite state representation with n states, the immage of X under the map $H : X \rightarrow \mathbb{R}^n$ given by $\Phi(A) = (\max_A u_1(), ..., \max_A u_n())$ is a full dimensional convex subset of $\mathbb{R}^n$. Let p be such that $\Phi(\{p\})$ is in the interior of $H[X]$.Then for any (element wise) small enough vector $\boldsymbol{\epsilon}$ in $\mathbb{R}^n$, we must have $\Phi(\{p\})+\boldsymbol{\epsilon} \in H[X]$. So let $\epsilon_i > 0$ for all i corresponding to negative states and $\epsilon_j \leq 0$ for all j corresponding to positive states. If $\succcurlyeq$ is non- monotonic, it has at least one negative state. Hence letting A be any set such that $\Phi(A) = \Phi(\{p\}) + \boldsymbol{\epsilon}$, we get that $\Phi(A \cup \{p\})_i = \max\{\Phi(A)_i, \Phi(\{p\})_i\} = \Phi(\{p\})_i$ for i corresponding to positive states and $\Phi(A \cup \{p\})_j > \Phi(\{p\})_j$ for j corresponding to negative states, which implies that $\{p\} \succ A \cup \{p\}$. ■

5. On the Dominance relation of Kreps (1979)

Kreps Dominance Relation. Kreps considers a complete transitive preference over menus of a finite set, and assumes:

1) Monotonicity: $A \subseteq B \Rightarrow B \succcurlyeq A$

2) Axiom (1.5) : $A \sim A \cup B \rightarrow A \cup C \sim A \cup B \cup C$.

He then defines a subrelation $\trianglerighteq$ over menus given by $A \trianglerighteq B$ if $A \succcurlyeq A \cup B$. Notice that $\trianglerighteq$ is reflexive, that by Monotonicity and (1.5) we have that $\trianglerighteq$ is transitive and moreover $\succcurlyeq$ is a proper completion of $\trianglerighteq$. Also notice the subrelation $\trianglerighteq$ can be defined by asking that

a) $\trianglerighteq \subseteq \succcurlyeq$, $\triangleright \subseteq \succ$.

b) $A \bowtie B \Leftrightarrow A \prec A \cup B \succ B$.

This definition highlights the nature of the $\trianglerighteq$ relation: assuming that one prefers flexibility only when unsure about the comparison between menus A and B, $\trianglerighteq$ expresses the part of your preference $\succcurlyeq$ that you are sure about. So another way of seeing the Kreps axioms is as ensuring that $\succcurlyeq$ can be seen as a completion of $\trianglerighteq$, where $\trianglerighteq$ is the underlying "sure" preference over sets and the proper completion $\succcurlyeq$ dubs $A \cup B$ strictly better than both A and B only when $\trianglerighteq$ is uncertain about their value.

A similar argument can be made in the menus of lotteries environment, for a preference satisfying Monotonicity and Independence instead. As it can be shown that Monotonicity and Independence imply assumption (1.5), the usual construction for the dominance relation will deliver a reflexive and transitive relation $\trianglerighteq$ whose completion is $\succcurlyeq$. Moreover $\trianglerighteq$ now satisfies Independence. This relation can also be uniquely identified using a) and b) as before.

Dropping Monotonicity. Now suppose we wish to drop Monotonicity, but keep Independence. Can we still obtain an underlying relation expressing those comparisons the DM is "sure of" such that any strict preference for flexiibility coincides with a pair where $\trianglerighteq$ is not sure? Now we cannot define it using $A \not\succcurlyeq A \cup B$, since, as we are not requiring monotonicity, we might very well have a case in which $A \succ A \cup B$. But we can still go the other way around: we can ask if there is a subrelation $\trianglerighteq$ of $\succcurlyeq$ satisfying Independence such that a) and b) holds. Our Proposition 1 says this can be done iff $\succcurlyeq$ is Monotonic. In fact our result is stronger than that. It holds even if we substitute b) with

$$A \bowtie B \Rightarrow A \preccurlyeq A \cup B \succcurlyeq B$$

So even if we allow our subrelation to only describe some of the comparisons I am sure of, there is no escape from Monotonicity.