Choice Deferral, Indecisiveness and Preference for Flexibility*

Leonardo Pejsachowicz[†] Severine Toussaert[‡]

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Preliminary

Indecision is the key to flexibility. Proverb

Abstract

We introduce a model of menu choice in which a person's psychological tastes and revealed preferences may be distinct. The former is modeled by means of a possibly incomplete (but otherwise rational) preference relation $\succcurlyeq$, and the latter by a completion $\succcurlyeq^*$ of that relation. The two relations are connected through an axiom which formalizes the following intuitive rule: "Whenever in doubt about which menu is better, don't commit now; just leave options open." The main result of this paper is that, under the usual assumptions of the menu choice literature, this deferral property alone forces $\succcurlyeq^*$ to exhibit a preference for flexibility, thus giving credence to the proverb above. In particular, we find that there is a fundamental tension between non-monotonic preferences, such as preferences for self-control, and tendency to defer choice due to indecisiveness.

KEYWORDS. Incomplete preferences, preference for flexibility, choice deferral.

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[†] *Department Of Economics, New York University.* Email: *lp961@nyu.edu.*

[‡] *Department Of Economics, New York University.* Email: *st1445@nyu.edu*

1. Introduction

The primitive of the theory of choice among opportunity sets is a preference relation defined on a collection X of subsets of a given space of alternatives. These subsets, called "menus," are generally interpreted as feasible sets from which an alternative will be selected at some later (unmodeled) stage. With this dynamic interpretation in mind, Kreps (1979) introduced in a seminal paper a behavioral property called "preference for flexibility," which is now a fairly common postulate in this literature.[1] According to this postulate, the preference relation of the decision maker ranks any superset of a menu at least as high as that menu. Clearly, this monotonicity property seems particularly meaningful if the decision maker aims to accommodate unforeseen contingencies. However, it is by no means unexceptionable. There are situations in which one may prefer smaller menus to larger ones. Indeed, an agent may well favor commitment if he fears to be tempted by some option as in the model of Gul and Pesendorfer (2001). Alternatively, he might prefer to restrict his choice set if he anticipates regret as in the model of Sarver (2008). In this respect, it seems important to take a closer look at the rationales behind preference for flexibility.[2]

The primary objective of this paper is to show that the inability of an agent to compare two menus at a basic (psychological) level may alone account for his preference for flexibility. The idea of indecisiveness is of course not new in decision theory; it dates back to Aumann (1962), and is captured by dropping the assumption of *completeness* of one's preferences. Moreover, while it is mostly studied in other contexts,[3] indecisiveness is likely to be more

[1]For instance, Nehring (1999) studies preference for flexibility in a Savagean context. Dekel et al. (2001, 2007) extend the work of Kreps (1979) to a lottery framework. Recently, Ahn and Sarver (2013) combined preference for flexibility ex ante with random choice ex post. This property has also been recently analyzed in a dynamic setting (see, for example, Krishna and Sadowski (2012)). The complete list of papers which work with this property is, however, too long to be mentioned here.

[2]One common justification for preference for flexibility stems from the idea that the decision maker may feel uncertain about what his future tastes will be, and indeed, the famous representation theorems of Kreps (1979) and Dekel et al. (2001) give substance to this interpretation. Yet, this is still an "interpretation", and as such, does not provide a justification for why larger menus may be ranked at least as high as their subsets, and sometimes strictly so.

[3]There is now a sizable literature on the theory of incomplete preferences. The recent contributions can be found on a large spectrum, ranging from the ordinal framework of

pronounced in the context of menu preferences. After all, one of the main justifications for allowing for indecisiveness stems from the complex nature of the objects of choice (such as goods with multiple attributes or lotteries with large support), which the decision maker may find hard to compare. This source of indecisiveness was originally noted by Aumann (1962) who wrote that some "... decision problems might be extremely complex, too complex for intuitive "insight", and our individual might prefer to make no decision at all in these problems." In this respect, one would expect the situation to be no easier when the objects of choice are themselves decision problems, as is the case in the menu choice framework. It thus appears that representing the psychological tastes of an agent by a potentially incomplete preference relation $\succcurlyeq$ on X is quite natural in the present setting. By $A \succcurlyeq B$, we understand that there is no doubt in the mind of the agent that the menu A is better than B. Nevertheless we leave open the possibility that neither $A \succcurlyeq B$ nor $B \succcurlyeq A$ may hold.

Yet, more often than not, we do not observe the inherent tastes of a decision maker; we rather see the choices he makes. Suppose that from the feasible set $\{A, B\}$ of menus, we see him choosing A instead of B. We then say that A is revealed preferred to B by this agent. Assuming that a choice has to be made at all times, this gives rise to a *complete* preference relation, say, $\succcurlyeq^*$, on X. If $A \succcurlyeq^* B$, we understand that the agent declared A better than B through his choice, but we do not know if $A \succcurlyeq B$ actually holds. Perhaps the agent was unable to compare A and B on the basis of his core preferences,[4] and his choice of A over B followed from the recommendation of a second party and/or from the adoption of some ad hoc choice procedure. By contrast, if $A \succcurlyeq B$ actually holds, we surely expect $A \succcurlyeq^* B$ to hold.

The literature on menu preferences usually takes the revealed (complete) preference relation $\succcurlyeq^*$ on X as the primitive of the analysis. In this paper, we instead take two preference relations $\succcurlyeq$ and $\succcurlyeq^*$ on X as the primitives.

Peleg (1975), Ok (2002) or Evren and Ok (2010) to the cardinal world of lotteries (Dubra et al. (2004) and Baucells and Shapley (2008)) or choice under uncertainty (Bewley (1986), Galaabaatar and Karni (2013), Ok et al. (2012)).

[4]This might be the case if for instance he cannot foresee how he will feel in the future, when choosing from the selected menu. Alternatively, his tastes might be so genuinely incomplete that he cannot make up his mind between some sets even when they contain a single alternative. The idea that indecisiveness may be understood either as incompleteness in beliefs (here about one's tastes tomorrow) or as incompleteness in tastes was recently formalized by Ok et al. (2012) in the Savagean context of uncertainty.

As discussed above, $\succcurlyeq$ is possibly incomplete and corresponds to the psychological tastes of the agent, while $\succcurlyeq^*$ is a completion of $\succcurlyeq$ and corresponds to the revealed preferences of the agent. Thus, when $\succcurlyeq = \succcurlyeq^*$, our setting reduces to the standard framework.[5]

This modeling approach allows us to explore the connections between an agent's indecisiveness and his preference for flexibility. On the one hand, indecisiveness is a psychological phenomenon, which is captured by the potential incompleteness of the core relation $\succcurlyeq$. On the other hand, preference for flexibility is a behavioral phenomenon, which is captured by the monotonicity of $\succcurlyeq^*$ (which formally means that $A \succcurlyeq^* B$ for any two menus A and B with $A \supseteq B$). Our main axiom builds a bridge between those two phenomena and allows us to investigate the implications of indecisiveness (of $\succcurlyeq$) for monotonicity (of $\succcurlyeq^*$). This axiom formalizes the following intuitive rule: "Whenever in doubt, just leave options open." More formally, this rule says that if A and B are incomparable according to $\succcurlyeq$, one would expect that $A \cup B \succcurlyeq^* A$ (but we do not know whether $A \succcurlyeq^* B$ or $B \succcurlyeq^* A$). Intuitively, an indecisive decision maker will often seek to defer choice if, for instance, he expects to be better informed in the future or simply needs additional time for contemplation about a difficult decision. Under such circumstances, choosing not to commit to a given menu can be seen as a cautious attitude. We thus call this property the *Cautious Deferral Axiom*.[6]

Our primary interest is to understand the behavioral consequences of the Cautious Deferral Axiom and its relation to monotonicity. In order to do so, we adopt the standard framework of the theory of menu preferences and impose the usual rationality assumptions of this literature (Section 3). Our main result (Section 2.4) highlights an interesting connection between Cautious Deferral and monotonicity. When the agent faces no internal conflict, that is, when $\succcurlyeq$ is complete, our rule imposes no restriction on choice behavior. On the other hand, we find that even a minimal amount of indecisiveness

[5]This way of modeling an individual with two relations, one being a completion of the other, has been an object of recent investigation. For instance, this approach has been adopted by Gilboa et al. (2010), Cerreia-Vioglio (2012), Cerreia-Vioglio et al. (2013), Lehrer and Teper (2013) or Nehring (2009) in a world of uncertainty and by Danan (2003) in a general menu choice setting. See Section 3 for more details.

[6]Note that this property is specific to the language of menus, which allows the formulation of the notion of "choice deferral" through the agent's preference for flexibility. For this reason, such a connection between a preference relation and its completion has not been explored in decision theory. One exception is Danan (2003), about which more will be said shortly.

has far-reaching consequences: Provided that $\succcurlyeq$ is not complete, $\succcurlyeq^*$ must exhibit preference for flexibility on its entire domain and this, regardless of the extent of the incompleteness of $\succcurlyeq$. In this sense, the Cautious Deferral Axiom provides foundations for preference for flexibility and gives credence to the proverb "Indecision is the key to flexibility."

One can also view our main result as an impossibility theorem (Section 2.5). To wit, consider an agent with incomplete tastes who satisfies the basic rationality postulates but may violate set monotonicity because, for instance, he suffers from temptation à la Gul and Pesendorfer (2001). Then our main theorem shows that if this agent also abides by the Cautious Deferral rule, temptation, self-control and flexibility motives all disappear. What we are left with is a standard decision maker who evaluates a set by its maximal elements with respect to some utility function. In this respect, we identify a basic tension in the menu choice framework between non-monotonic preferences, indecisiveness and the Cautious Deferral Axiom.

2. The Model

2.1. Preliminaries

We work in the standard framework of the theory of menu preferences as in, say, Dekel et al.(2001) (from now on DLR). In what follows, we let Δ stand for the set of all probability distributions (lotteries) over a prize space of finite cardinality n. The generic members of Δ are denoted as p, q, r, ... etc. We let X stand for the set of all nonempty closed subsets of Δ. As is standard in this literature, we view X as a metric space relative to the Hausdorff metric. The generic members of X are denoted as A, B, C, ... etc., and are referred to as *menus*. These sets are interpreted as opportunity sets from which the decision maker will choose an option at a later (unmodeled) stage.

In what follows, we also consider X as an algebraic entity by imposing on it the mixture operation induced by the Minkowski sum of sets. That is, for any A and B in X and any $0 \leq \lambda \leq 1$, we define

$$\lambda A + (1 - \lambda)B := \{\lambda p + (1 - \lambda)q \mid p \in A \text{ and } q \in B\}$$

which is itself an element of X.

A binary relation $\succcurlyeq$ on X is said to be a *preorder*, or a preference relation, on X if it is reflexive and transitive. The asymmetric part of this relation is denoted by $\succ$, that is, we have $A \succ B$ if and only if $A \succcurlyeq B$ but not $B \succcurlyeq A$. (The symmetric part of $\succcurlyeq$, denoted by $\sim$, is thus $\succcurlyeq \setminus \succ$.) A preorder $\succcurlyeq$ on X is said to be *monotonic* if $A \succcurlyeq B$ holds for every A and B in X with $A \supseteq B$.

We denote the non-comparability part of a preorder $\succcurlyeq$ by $\bowtie$. That is, $\bowtie$ is the binary relation on X such that $A \bowtie B$ if and only if neither $A \succcurlyeq B$ nor $B \succcurlyeq A$. If $\bowtie = \varnothing$, then $\succcurlyeq$ is said to be *complete*. In turn, given a preference relation $\succcurlyeq$ on X, by a *completion* of $\succcurlyeq$, we mean a complete preference relation $\succcurlyeq^*$ such that

$$A \succcurlyeq B \text{ implies } A \succcurlyeq^* B \quad \text{and} \quad A \succ B \text{ implies } A \succ^* B$$

We say that an ordered pair $(\succcurlyeq, \succcurlyeq^*)$ is a *preference structure* on X if $\succcurlyeq$ is a preference relation on X and $\succcurlyeq^*$ is a completion of $\succcurlyeq$. In this paper, we will consider such structures as the primitives of the environment.[7] Of course, when $\succcurlyeq = \succcurlyeq^*$, the induced preference structure can be identified with a complete preference relation, so the standard framework of menu preferences is a special case of ours.

We interpret a preference structure $(\succcurlyeq, \succcurlyeq^*)$ as follows. The first relation $\succcurlyeq$ represents the decision maker's psychological tastes. Simply put, this core relation captures among which menus the agent is completely decisive. In this respect, $\succcurlyeq$ is unobservable to the modeler. On the other hand, the second relation $\succcurlyeq^*$ stems from the revealed preferences of the agent through his choice behavior. As we presume that we observe *all* choices of the agent across pairwise problems, this relation is taken as complete. Furthermore, it is consistent with the core relation of the agent, that is, whenever $\succcurlyeq$ ranks menus in a particular manner, $\succcurlyeq^*$ does so in exactly the same way.

2.2. Basic Axioms

Throughout the paper, we will work with a preference structure $(\succcurlyeq, \succcurlyeq^*)$ on X which is rational in the standard sense. That is, we will impose the following two axioms on $(\succcurlyeq, \succcurlyeq^*)$:

[7] We note that such preference structures have been previously studied by Danan (2003), a discussion of which is provided in Section 3.

Axiom 1 (Independence): For every $A, B, C \in X$ and $0 < \lambda < 1$, we have

$$A \succcurlyeq B \quad \text{if and only if} \quad \lambda A + (1-\lambda)C \succcurlyeq \lambda B + (1-\lambda)C$$

and similarly for $\succcurlyeq^*$.

Axiom 2 (Continuity): Both $\succcurlyeq$ and $\succcurlyeq^*$ are closed in $X \times X$.[8]

As these assumptions are standard in the literature on menu preferences, we do not discuss their motivation here.[9]

In what follows, it will be necessary to impose an additional, and somewhat technical, assumption on the second relation $\succcurlyeq^*$ of a preference structure on X. The following auxiliary definition facilitates the statement of that property.

Definition: Let A be an element of X and $\succcurlyeq^\circ$ a preorder on X. We say that a subset A_0 of $\text{conv}(A)$ is $\succcurlyeq^\circ$-*critical* for A if $B \sim^\circ A$ for every $B \in X$ such that $A_0 \subseteq \text{conv}(B) \subseteq \text{conv}(A)$.[10]

To get an intuition for this definition, take any two convex sets A and B in X such that $A_0 \subset B \subset A$, where A_0 is a $\succcurlyeq^\circ$-critical subset of A. We must then have $A_0 \sim^\circ B \sim^\circ A$. . Put differently, the additional information contained in $B \setminus A_0$ is irrelevant in the evaluation of A. Therefore, a critical subset of A can be thought of as extracting all the essential information contained in A.

The assumption we need, which is called the Finiteness Axiom in the related literature, was first introduced in Dekel et al.(2009). This postulate, and some of its analogs, are adopted in many works on menu preferences.[11] We impose this property on a preference structure $(\succcurlyeq, \succcurlyeq^*)$ on X, but note

[8] $X \times X$ is viewed here as the product metric space induced by the Hausdorff metric on X.

[9] See DLR and Gul and Pesendorfer (2001) for discussions of Axiom 1 in the context of menu preferences, and Evren and Ok (2011) for that of Axiom 2 in the context of arbitrary (potentially incomplete) preferences relations.

[10] Here $\text{conv}(A)$ denotes the convex hull of A.

[11] See, for instance, Stovall (2010), Riella (2013), and Ahn and Sarver (2013) for recent papers which use the Finiteness Axiom. Kopylov (2009b) uses a different axiom which applies to a more general setting, but one that is equivalent to the Finiteness Axiom for a complete and continuous preference relation on X that satisfies the Independence Axiom.

that it constrains only the second relation $\succcurlyeq^*$ of this structure.

Axiom 3 (Finiteness): Every menu in X has a finite $\succcurlyeq^*$-critical subset.

Given the interpretation of critical subsets of menus, what this postulate says is that the problem of evaluating any given menu can always be reduced to evaluating a *finite* subset of that menu. As is the case for Dekel et al. (2009) it is imposed here in order to simplify the analysis. In particular, it allows us to use finite-dimensional techniques when studying the structure of $(\succcurlyeq, \succcurlyeq^*)$.[12]

2.3. Cautious Deferral

We now introduce our main axiom, called the *Cautious Deferral Axiom*, which connects the preference relations $\succcurlyeq$ and $\succcurlyeq^*$.[13]

Axiom 4 (Cautious Deferral): For every A and B in X,

$$A \bowtie B \text{ implies } A \cup B \succcurlyeq^* A$$

Note that one can always view an incomplete preference $\succcurlyeq$ as the intersection of a collection of complete preferences, each of which represents a different criterion of evaluation in the mind of the agent.[14] Thus, the decision maker's inability to compare two menus through $\succcurlyeq$ can be seen as stemming from the conflict between the various considerations that may enter his evaluation of the problem, or equivalently, between his different "selves" where the tastes of each self are represented by a complete preference relation on X. For instance, consider an agent who must decide between two restaurants

[12]Unfortunately, we do not know at present if our main theorem can be obtained without Axiom 3. We note that Chatterjee and Krishna (2011) show that differences exist between the behavioral features captured by models that do satisfy the Finiteness Axiom and those that do not. This suggests exercising some caution in the interpretation of our results.

[13]Although very different in their structure and context, several papers present similarities with the approach adopted here by considering a pair of preference relations connected through some behavioral axiom such as our Cautious Deferral rule. For instance in the Anscombe-Aumann framework, Gilboa et al. (2010) connect two preference relations, one incomplete and one complete, with an axiom called Caution which captures the agent's behavioral attitude towards uncertainty. That paper, as well as other papers which adopt such an approach, are discussed in Section 3.

[14]This is a set-theoretic fact; see Section 1.4 of Ok (2007) for more details.

A and B and evaluates each menu according to how healthy and how appetizing the available options are in each menu. If menu A contains the most healthy option (option a) and menu B contains the most appetizing option (option b), the agent might be unable to decide at which restaurant to dine. This inability to compare two menus A and B is captured in our model by setting $A \bowtie B$.

When intimately torn between two menus A and B, what rule of conduct may the decision maker adopt? If the agent is constrained to choose from the feasible set $\{A, B\}$, our model remains silent on what the final choice will be. All we know is that $A \succcurlyeq^* B$ if this choice is A, and similarly $B \succcurlyeq^* A$ if B is selected. (Recall that $\succcurlyeq^*$ stands for the *revealed* preference of the agent.) As $A \bowtie B$, we also do not have any rationale for why either of these choices might occur. (For instance, the agent may have consulted a second party who suggested $A \succcurlyeq^* B$ or he may have adopted some ad hoc choice procedure.) But now suppose that his choice set is actually $\{A, B, A \cup B\}$. In the context of the example above, our agent may have the additional option of going to a restaurant with a larger menu where the options of both restaurants A and B are available. Alternatively, one might think of $A \cup B$ as waiting until dinner time in order to pick a restaurant rather than making a reservation at either A or B. In either case, it stands to reason that the decision maker would seize this additional opportunity, which would mean $A \cup B \succcurlyeq^* A$ (and by symmetry, $A \cup B \succcurlyeq^* B$), since this leaves all options open at the later stage: by not committing to either menu, deferral is a cautious attitude for the agent. This suggests that perhaps we have $A \cup B \succcurlyeq A$ and therefore, $A \cup B \succcurlyeq^* A$, meaning that the agent intimately prefers the flexibility offered by the wider menu. Or perhaps the agent is still indecisive between A and $A \cup B$, that is, $A \bowtie A \cup B$.[15] Even in this case, it seems that committing today to A as opposed to $A \cup B$, that is, $A \succ^* A \cup B$, is unwarranted. For instance, suppose that $\succcurlyeq^*$ results from the advice of a second party. Then a natural recommendation of this party would be "if you are not sure what menu to commit to, choose not to commit at all and wait if this is possible".

[15] To illustrate this possibility, consider an agent who has two "selves". One of these "selves" has self-control preferences à la Gul and Pesendorfer (2001) and assesses the value of a menu A as $W(A) = \max_{a \in A}[u(a) + v(a)] - \max_{a \in A} v(a)$, while the other evaluates A as a weighted average of the "commitment" and "temptation" utilities, that is, $V(A) = p \max_{a \in A} u(a) + (1-p) \max_{a \in A} v(a)$, where p is a real number in $(0, 1)$. Now assume that the agent needs to choose between $\{a\}$ and $\{a, b\}$, and that $u(a) > u(b)$ while $v(b) > v(a)$. Then, we have $\{a\} \bowtie \{a, b\}$ because $W(\{a\}) > W(\{a, b\})$ and $V(\{a, b\}) > V(\{a\})$.

This suggests again $A \cup B \succcurlyeq^* A$, which is exactly what the Cautious Deferral Axiom says.

This discussion points to the intuitive nature of the Cautious Deferral Axiom, but it does not say anything about why a conflicted decision maker may find choice deferral desirable. In our view, this may happen for at least two reasons. On the one hand, an indecisive agent might prefer to postpone his choice if he expects his internal conflict to be resolved in the future. For instance, the agent in our example might later find himself in an indulgent mood, thus making restaurant B a clearly more attractive choice than A. In this respect, choice deferral may have an informational value to the decision maker. On the other hand, even when the decision maker does not expect his indecisiveness to resolve in time, choice deferral might be valued if it helps one to come to terms with a difficult decision. By leaving options open, the agent allows himself additional time for contemplation about this difficult choice problem.[16]

In passing, we emphasize that the Cautious Deferral Axiom has a positive, rather than a normative, content. First, introspection and anecdotal evidence seem to provide support for this intuitive rule.[17] Furthermore, several empirical studies attest to the descriptive appeal of this axiom by documenting a link between choice deferral and the inability to compare alternatives (Tversky and Shafir (1992), Dhar (1997), Tykocinski and Ruffle (2003); see Section 3 for more details)

2.4. Main Results

The main objective of the present work is to understand the implications of the Cautious Deferral Axiom with respect to the preference for flexibility that may be exhibited by the choice behavior of a decision maker. More precisely, we wish to understand the extent to which the behavioral preference relation $\succcurlyeq^*$ may exhibit a preference for flexibility when the preference structure

[16] A dramatic illustration of this case can be found in the novel of William Styron, *Sophie's Choice.* One can think that the choice Sophie faced between saving her son from the gas chamber or saving her daughter might have been anything but a spontaneous decision.

[17] Tversky and Shafir (1992) report an anecdote that was shared by Thomas Schelling. The latter had decided to buy an encyclopedia for his children but, to his discontent, he found two in the bookstore. Even though both options seemed satisfactory, the difficulty to make a choice between the two encyclopedias led Schelling to buy neither.

$(\succcurlyeq, \succcurlyeq^*)$ on X satisfies Axiom 4. It is plain that the answer depends on how incomplete the psychological preference relation $\succcurlyeq$ is. At one extreme of the spectrum is the case where this relation is complete, that is, $\bowtie = \varnothing$. In this case, Axiom 4 becomes a triviality, yielding no clue as to the structure of $\succcurlyeq^*$. At the other extreme is the case where $\succcurlyeq$ is unable to rank any two distinct menus, that is, $\bowtie = \{(A, B) : A \neq B\}$. In that case, it is readily verified that Axiom 4 implies the monotonicity of $\succcurlyeq^*$, that is, $A \succcurlyeq^* B$ for every A and $B \in X$ such that $A \supseteq B$. Therefore, while the Cautious Deferral Axiom enforces some monotonicity on $\succcurlyeq^*$, the scope of this monotonicity, and in particular, whether or not $\succcurlyeq^*$ is monotonic on its entire domain, depends on the degree of indecisiveness exhibited by the core preference relation $\succcurlyeq$.

The main result of this paper shows that, in the context of *rational* preferences structures on X, the restrictions on $\succcurlyeq^*$ imposed by the Cautious Deferral Axiom are actually much stronger than the previous discussion suggests. Indeed, for preference structures $(\succcurlyeq, \succcurlyeq^*)$ that satisfy Axioms 1-3 as well as the Cautious Deferral Axiom, it turns out that the full completeness of $\succcurlyeq$ is the only way in which $\succcurlyeq^*$ may escape from exhibiting a global preference for flexibility. Even when the psychological preference relation $\succcurlyeq$ presents only a minimal amount of incompleteness, that is, whenever $\bowtie \neq \varnothing$, it turns out that $\succcurlyeq^*$ is sure to be monotonic on its entire domain. Under the usual rationality postulates, the Cautious Deferral Axiom thus implies preference for flexibility so long as the core preferences $\succcurlyeq$ of the individual are incomplete, and this, regardless of the extent of the incompleteness of $\succcurlyeq$.

THEOREM 1: Let $(\succcurlyeq, \succcurlyeq^*)$ be a preference structure on X which satisfies Axioms 1-4. Then, either $\succcurlyeq$ is complete or $\succcurlyeq^*$ exhibits preference for flexibility.

On the one hand, this theorem shows that there is a fundamental tension between non-monotonic preferences over menus, incompleteness of one's tastes and the Cautious Deferral Axiom. In particular, an agent with incomplete tastes cannot exert commitment on *any* part of the menu space while at the same time completing his core preferences in concert with the Cautious Deferral Axiom. On the other hand, this theorem appears to provide a psychological foundation for the property of preference for flexibility. Insofar as one concedes that a rational decision maker may sometimes be indecisive and also that her revealed preferences are consistent with the Cautious Deferral Axiom, then those preferences must exhibit preference for flexibility in all contingencies. In accordance with the opening vignette of the paper, we thus find that indecisiveness (on the part of the psychological preferences $\succcurlyeq$) is the key to preference for flexibility (on the part of the revealed preference $\succcurlyeq^*$) for rational individuals who behave consistently with the Cautious Deferral Axiom.

We should reiterate here that this is a consequence of our strong rationality assumptions. In particular, although Axiom 1 is generally presented as the natural counterpart in a menu choice setting of the standard Expected Utility axiom, it has been argued that the affinity it implies may be too strong in this particular setting[18]. One might thus interpret our result as an additional argument against the version of the Independence Axiom we adopted here: if we wish to allow for both preference for commitment and preference for flexibility, while retaining the Cautious Deferral Axiom and its descriptive content, we have to make do without affinity in Minkowsky mixtures.

A natural question at this point is if the core relation $\succcurlyeq$ is also forced to be monotonic (when it is incomplete) under the conditions of Theorem 1. We show next by an example that this need not be the case.

Example: Consider the real maps U and V on X defined by

$$U(A) := \lambda \max_{p \in A} u(p) - \max_{p \in A} v(p)$$

and

$$V(A) := \lambda \max_{p \in A} v(p) - \max_{p \in A} u(p),$$

where u and v are distinct continuous and affine functions on $\triangle$ and $\lambda > 1$. Then, it can easily be seen that the relation $\succcurlyeq$ defined as

$$A \succcurlyeq B \text{ if and only if } U(A) \geqslant U(B) \text{ and } V(A) \geqslant V(B),$$

is incomplete and satisfies Axioms 1 - 3. On the other hand, the preference relation $\succcurlyeq^*$ on X represented by $U + V$ is a completion of $\succcurlyeq$ which satisfies

[18] For example, Marinacci et al.(2007) propose that, when the decision maker has a coarse perception of future contingencies, he will only satisfy a weaker assumption, which is an adaptation of the Uncertainty Aversion Axiom to the menu choice setting. On the side of the temptation literature Noor and Takeoka (2010a) and (2010b) propose models à la Gul and Pesendorfer in which the cost of resisting temptation is either convex or menu dependent, which causes a violation of the affinity assumption.

the Cautious Deferral Axiom. Therefore, in line with our main theorem, $\succcurlyeq^*$ exhibits preference for flexibility but $\succcurlyeq$ is not monotone on X.

It may be worth noting that the monotonicity of $\succcurlyeq$ can be ensured in Theorem 1 if in the statement of the Cautious Deferral Axiom, preference for flexibility is instead imposed directly on this core relation and if, in the statement of Finiteness, the relation $\succcurlyeq$ is substituted for $\succcurlyeq^*$[19] . To make this precise, consider the following modified versions of the Finiteness and Cautious Deferral Axioms:

Axiom 3b ($\succcurlyeq$-Finiteness): Every menu in X has a finite $\succcurlyeq$-critical subset.

Axiom 4b ($\succcurlyeq$-Cautious Deferral): For every A and B in X such that $A \nsubseteq B$ and $B \nsubseteq A$,[20]

$$A \bowtie B \text{ implies } A \cup B \succcurlyeq A.$$

We have the following counterpart to Theorem 1:

Theorem 2: Let $\succcurlyeq$ be a preference relation on X which satisfies Axioms 1, 2 and Axioms 3b, 4b. Then, $\succcurlyeq$ is either complete or it exhibits preference for flexibility.

This result follows from a non-trivial modification of the proof of our main result. For brevity, we will omit the details of this proof (which is available from the authors upon request).

2.5. Self-Control versus Cautious Deferral

One way of interpreting Theorem 1 is as a negative result, which shows that non-monotonic preferences must be renounced if one wishes to maintain indecisiveness together with Cautious Deferral. To demonstrate this point, consider the Gul and Pesendorfer (2001) model of temptation and self-control.

[19]Notice that if Finiteness is imposed on $\succcurlyeq$, it will also hold for $\succcurlyeq^*$, thus here we are strengthening Axiom 3

[20]The proviso that A and B be incomparable with respect to inclusion is only to guarantee that the axiom is meaningful for all sets; this proviso does not play a role in the main argument of the proof.

This model takes as a primitive the revealed preference relation $\succcurlyeq^*$ on X, and besides the standard rationality assumptions, imposes the Set Betweenness Axiom. This axiom states that $A \succcurlyeq^* B$ implies $A \succcurlyeq^* A \cup B \succcurlyeq^* B$, for any two menus A and B. To understand the first part of the implication, consider an agent for whom $A \succ^* B$ because the best alternative in A is strictly preferred to the best alternative in B. Then, the axiom allows for the agent to strictly prefer committing to A today rather than to $A \cup B$, that is, $A \succ^* A \cup B$, which is rather reasonable if B contains a temptation that the agent wishes to avoid.

Even without appealing to the representation theorem of Gul and Pesendorfer (2001), we can show that the Cautious Deferral Axiom goes against the agent's desire for commitment. To this end, suppose that the core preferences of the agent are represented by some incomplete preference relation $\succcurlyeq$, and $\succcurlyeq^*$ is a completion of $\succcurlyeq$. Now take any two menus A and B. If $A \succcurlyeq B$, then the motivation behind the Set Betweenness Axiom applies, so we may comfortably posit that $A \succcurlyeq A \cup B$. But suppose that $A \bowtie B$ is the case, that is, the agent is unable to decide (today) between A and B on the basis of his core preferences. Then, the Cautious Deferral Axiom tells us that the agent's fear of making a mistake by committing to either A or B would overwhelm any other consideration, prompting him to set $A \cup B \succcurlyeq^* A, B$. Although this might seem to be a strong assumption, at an intuitive level this behavior seems to make perfect sense[21]: the agent exerts self-control when he can clearly identify the elements of temptation that may be present (i.e. $A \succcurlyeq B$ implies $A \succcurlyeq A \cup B$ and hence $A \succcurlyeq^* A \cup B$), but wishes not to commit when comparing two menus between which he is indecisive (i.e. $A \bowtie B$ implies $A \cup B \succcurlyeq^* A$).

This intuition runs, however, to a severe difficulty, at least in the presence of the standard rationality axioms. For, take any two menus A and B such that $A \succcurlyeq^* B$. By the Set Betweenness Axiom, it must be that $A \succcurlyeq^* A \cup B$.

[21] To see why the assumption might be strong, consider an agent who evaluates sets by combining the value of the best normative choice and that of the strongest temptation. Suppose he faces the options: broccoli b, potato chips p and chocolate cake c. Potato chips and chocolate cake are more tempting than broccoli but the chips are more tempting as a salty craving, while the cake is more tempting as a sweet craving. On the other hand, broccoli is healthier than either of the options. If the agent is unable determine which is stronger between the salty and the sweet craving, it will be the case that $\{b, p\} \bowtie \{b, c\}$. On the other hand, it makes sense that he would always choose either $\{b, p\}$ or $\{b, c\}$ over $\{b, c, p\}$, since the latter set guarantees the worst temptation. Such agent would clearly violate Cautious Deferral (we thank John Stovall for providing this example).

If, however, $\succcurlyeq^*$ is also a completion of $\succcurlyeq$, and both relations satisfy all the standard axioms[22] and the Cautious Deferral Axiom, then we know by Theorem 1 that $\succcurlyeq^*$ must be monotonic on its entire domain, so that $A \cup B \succcurlyeq^* A$. Therefore, $A \succcurlyeq^* B$ implies $A \sim^* A \cup B$, that is, our agent must be a standard decision maker who evaluates a menu by its best elements. Put differently, self-control motives are completely annihilated in the presence of the Cautious Deferral Axiom.

2.6. Sketch of the Proof

A complete binary relation on X is said to have a Finite Additive Expected Utility (FAEU) representation (first introduced and axiomatized in Dekel et al.(2009)) if it can be represented by a real function W on X defined by

$$W(A) = \sum_{i \in P} U_i(A) - \sum_{j \in N} V_j(A) \tag{2.1}$$

where

$$U_i(A) = \max_{p \in A} u_i(p) \quad \text{and} \quad V_j(A) = \max_{p \in A} v_j(p)$$

with the u_i's and v_j's being real-valued continuous and affine functions over Δ and P and N being finite index sets. Notice that the functional W is affine, and that each U_i is also a function of form (2.1) with $N = \varnothing$ and $|P| = 1$. A possible interpretation of the above representation is the following: there are $|N| + |P|$ equally likely subjective "states of the world" that the agent foresees, each corresponding to a particular realization of his future preferences over lotteries, identified by one of the functions in $\{u_i\}_{i \in P} \cup \{v_j\}_{j \in N}$. The agent evaluates positively his expected choices in states corresponding to indexes in P, and negatively his expected choices in states corresponding to indexes in N. A consequence of the FAEU representation is that a necessary condition for the agent to value the additional flexibility given by the menu $A \cup B$ over menu B is that there be at least one $i \in P$ for which $U_i(A) \geqslant U_i(B)$.

[22] In passing, we note that if $\succcurlyeq^*$ is represented by a self-control utility à la Gul and Pesendorfer (2001), $\succcurlyeq^*$ must satisfy all our standard axioms, including the Finiteness axiom. In fact, Dekel et al.(2009) show that the representation of Gul and Pesendorfer (2001) belongs to the class of Finite Additive Expected Utilities (FAEU) with one positive and one negative state; see the next section for a definition of FAEU and its interpretation.

The first step of our proof, which relies on results from Dekel et al.(2009) and Galaabaatar (2011), is to show that if the preference structure $(\succcurlyeq, \succcurlyeq^*)$ satisfies Axioms 1-3 then (i) The core relation $\succcurlyeq$ can be expressed as the intersection of a collection of relations $\{\succcurlyeq_k\}_{k\in\mathcal{K}}$ each of which is represented by a continuous and affine (w.r.t. Minkowski mixtures) function W_k; and (ii) The completion $\succcurlyeq^*$ has a FAEU representation $W^* = \sum_{i\in P^*} U_i^* - \sum_{j\in N^*} V_j^*$. We focus on the nontrivial case in which $\succ\neq\varnothing$ and $P^*\neq\varnothing$[23]. The following step, which is the crucial one, is to show that, if the pair $(\succcurlyeq, \succcurlyeq^*)$ also satisfies the Cautious Deferral Axiom, each non constant W_k can be expressed as a linear combination of W^* and its "positive states" $\{U_i^*\}_{i\in P^*}$. The argument is as follows. Since it must be that $U_i^*(A) \geqslant U_i^*(B)$ for some i for W^* to rank $A\cup B$ above B, we know that whenever for two sets A, B we have $U_i^*(B) > U_i^*(A)$ for all $i \in P^*$, it will be the case that $W^*(B) \geqslant W^*(A\cup B)$. If, in addition, $W^*(A) > W^*(B)$, the relation $\succcurlyeq^*$ will surely declare $A \succ^* A\cup B$. The Cautious Deferral Axiom then implies that we cannot have $A \bowtie B$ in any of these cases, which can be shown with some additional work to imply that each W_k must rank A strictly above B. Thus, for each $k\in\mathcal{K}$, we must have $W_k(A) > W_k(B)$ whenever $W^*(A) > W^*(B)$ and $U_i^*(B) > U_i^*(A)$ for all $i \in P^*$. In other words, each W_k is increasing in the Weak Pareto order induced by the collection $\{W^*, -U_1^*, ..., -U_P^*\}$. Since we are assuming $P^*\neq\varnothing$, if W^* is not monotone the above Weak Pareto order ranks at least two distinct menus. Consequently, we can use a Harsanyi type aggregation theorem to express each W_k as a non negative linear combination of W^* and each of the $-U_i$'s. This finding also allows us to show, using a separation argument, that W^* must lie in the relative interior of the cone $\mathcal{W}$ spanned by the W_k's. Finally we can use the above to derive the main result by contradiction. On the one hand, W^* is in the relative interior of $\mathcal{W}$. On the other, each W_k must lie in the cone spanned by W^* and all the $-U_i^*$'s. It can then be shown that only two configurations are possible. In one case, $W_k = W^*$ for all k, which implies that $\succcurlyeq$ is complete. In the other case, when $W_k \neq W_{k'}$ for some $k\neq k'$, each W_k must lie in the cone spanned only by the $-U_i$'s. Thus every W_k, and as a consequence also W^*, has a FAEU representation with $P^*=\varnothing$, a possibility that was already discarded.

[23] Indeed if $\succ = \varnothing$, monotonicity of $\succcurlyeq^*$ is an immediate consequence of Axiom 4. If $\succ\neq\varnothing$ but $P^*=\varnothing$, a continuity argument shows that the Cautious Deferral Axiom would be violated.

3. Related literature

There are a number of recent papers that investigate the completion and/or extension rules for incomplete preference relations in a variety of contexts. In particular, Gilboa et al. (2010) and Kopylov (2009a) investigate such completions in the Anscombe-Aumann framework. The former builds a bridge between the classical models of Bewley (2002) and Gilboa and Schmeidler (1989) while the latter ties Bewley (2002) to a model of ambiguity aversion known as the ϵ-contamination model. Similarly, Cerreia-Vioglio et al. (2013) use a completion rule for a preference relation over risky prospects in order to obtain a representation for a particular class of complete risk preferences. Finally, Lehrer and Teper (2013) consider a binary relation that is complete on a convex subset of the space of Anscombe-Aumann acts and propose an extension rule that generates Bewley (2002) preferences as well as a completion rule that generates the MaxMin preferences. It is worth noting that most of these papers adopt a notion of completion which is weaker than the one we adopt in this paper, for they do not require the strict part of the incomplete relation to be preserved by its completion. This weaker notion of a completion is not suitable to our purposes because we interpret the underlying incomplete preference relation $\succcurlyeq$ as representing the basic psychological tastes of the decision maker. In this respect, the revealed preferences of the agent, which represent a "completion" of $\succcurlyeq$, must be perfectly in line with his psychological tastes.

More closely related to the present work is that of Danan (2003) who also works with a pair of preferences ($\succcurlyeq$, $\succcurlyeq^*$) defined over a generic set of menus (which may or may not consist of lotteries). The interpretation of this structure is the same as ours, that is, $\succcurlyeq$ stands for the psychological preferences of the agent while $\succcurlyeq^*$ refers to his behavioral (revealed) preferences. The main objective of that paper is however different from ours, namely, finding conditions under which $\succcurlyeq$ can be identified through the observation of $\succcurlyeq^*$. One of Danan's main identifying conditions, which may appear as being quite similar to our Cautious Deferral axiom, can be expressed as: $A \bowtie B$ if and only if $A \cup B \succ^* A$ and $A \cup B \succ^* B$. This condition is however much stronger than the Cautious Deferral Axiom for the following two reasons. First, preference for flexibility is required to be strict while our axiom also allows for indifference. For instance, consider a decision maker who cannot compare any two distinct menus (perhaps due to his lack of information and/or interest) and hence is indifferent between any two menus. The preference structure of this agent is $(\succcurlyeq, X \times X)$, where $\bowtie = \{(A, B) : A \neq B\}$; it obviously satisfies the Cautious Deferral Axiom but fails Danan's condition. Secondly, Danan *identifies* incomplete preferences with a strict desire for flexibility by requiring the implication to go in both directions. This identification strategy is used in Danan and Ziegelmeyer (2006) in order to test the completeness axiom in a setting involving choices among menus of lotteries. In contrast, we do not impose the "if" part of Danan's axiom which is fairly restrictive. For instance, consider an agent with complete DLR preferences such that $A \cup B \succ^* A$ for at least two sets A and B. While these preferences are standard in the literature on menu preferences, they do not satisfy Danan's axiom. However, when $\succcurlyeq$ is complete, the Cautious Deferral Axiom is trivially satisfied.

In a different setting, Arlegi and Nieto (2001) also adopt a connecting condition which presents similarities with the Cautious Deferral Axiom. Their condition, called Restricted Monotonicity, connects an asymmetric binary relation P defined on a finite set X of alternatives to a binary relation $\succcurlyeq$ defined on the set of all menus from X. This condition requires that for any two alternatives x and y, xPy implies $\{x, y\} \sim \{x\}$ and not xPy implies $\{x, y\} \succ \{x\}$. If we identify xPy with $\{x\} \succ \{y\}$, the second part of their axiom can be seen as a stronger version of Axiom 4b when restricted to singleton sets (for, as in Danan (2003), preference for flexibility in the Restricted Monotonicity Axiom is required to be strict). Furthermore, we remain silent on what happens when two sets are comparable (the first part of that axiom); in particular, we do not impose that $A \succcurlyeq B$ implies $A \sim^* A \cup B$, the natural analog in our framework.

Finally, we note that several papers in the literature explore the connections between the internal conflict of a person and her tendency to defer choice. On the theoretical side, Gerasimou (2012) considers a model in which choice deferral is driven by incomparability. This idea is captured by a choice correspondence which can be empty-valued. Closely related is the work of Buturak and Evren (2010) who study the choice deferral phenomenon in a risky setting; they obtain a representation in which the decision-maker defers his choice from a set if and only if the option value of deferring (as an expected utility over subjective states) is larger than the current utility from the best available alternative in that set.[24]

[24]While the choice correspondence in Buturak and Evren (2010) takes a specific value

On the empirical side, Tversky and Shafir (1992) show in one study that subjects are more likely to wait when the objects of choice are difficult to compare than when one of the alternatives appears to be clearly superior on all attributes. In a related study, they find that subjects' propensity to wait in order to learn more about the various options available tends to increase if their choice is between two equally attractive options instead of just one option. In another study, Dhar (1997) finds that choice deferral tends to increase when the differences in attributes among the available alternatives are small, as this makes alternatives harder to distinguish. Finally, Tykocinski and Ruffle (2003) show that subjects may choose to postpone their choice even when they do not expect to receive additional information relevant to their choice, for instance if they need additional time for contemplation. Furthermore, they find a higher propensity to wait among subjects who express low confidence in committing themselves to a specific choice.

4. Conclusion

In the standard menu choice setting of DLR (2001), we discover a surprising connection between incompleteness and choice deferral on the one hand, and preference for flexibility on the other. We study the relationship between indecisiveness and choice deferral by modeling a decision maker with two preference relations. The first relation $\succcurlyeq$, possibly incomplete, represents the core preferences of the agent. The second relation $\succcurlyeq^*$ is a completion of $\succcurlyeq$ which represents the revealed preferences of the decision maker. The two relations are tied together through a new behavioral axiom called Cautious Deferral. This axiom captures the intuitive idea that "whenever in doubt, don't commit now, just leave options open" by requiring that whenever two menus A and B are incomparable according to the core relation $\succcurlyeq$, the agent should choose the union of the two, $A \cup B \succcurlyeq^* A$, if he has the possibility to do so. We analyze the consequences of this axiom for the preference for flexibility exhibited by $\succcurlyeq^*$.

In the context of rational preferences, we find that the Cautious Deferral Axiom has far-reaching consequences. As long as the core relation $\succcurlyeq$ is incomplete, the behavioral relation $\succcurlyeq^*$ is sure to be monotonic on its en-

when deferral is chosen, the standard rationality axioms are only imposed in situations where the decision maker does not defer. In this respect, their approach is equivalent to the empty-set approach adopted by Gerasimou (2012).

tire domain. The present work thus provides psychological foundations for the property of preference for flexibility, a property which has been commonly assumed in the literature on menu preferences. Alternatively, we find that there is a fundamental tension between non-monotonic preferences over menus, incompleteness of one's tastes and the Cautious Deferral Axiom.

A. Appendix

Part 1: Preliminaries Here we collect some results that will be useful for the proof of the main theorem. To begin with, following DLR, let

$$\mathcal{S} = \{s \in \mathbb{R}^n \mid \sum_i s_i = 0,\ \sum_i s_i^2 = 1\}.$$

be the set of normalized expected utilities on Δ. Each vector $s \in \mathcal{S}$ induces a different expected utility preference on Δ, and every nontrivial expected utility preference on Δ can be induced by exactly one vector $s \in \mathcal{S}$.

For any $A \in X$, let $\sigma_A : \mathcal{S} \to \mathbb{R}$ be given by $\sigma_A(s) = \max_{p \in A} s \cdot p$. We call σ_A the *support function* of A. It can be shown that support functions satisfy the following properties:

Lemma 1. *The map $\sigma : X \to \mathbb{R}^{\mathcal{S}}$ is continuous in the Hausdorff metric. Moreover, for all $A, B \in X$, we have:*

(i) $\sigma_{\alpha A + (1-\alpha)B} = \alpha\sigma_A + (1-\alpha)\sigma_B$.

(ii) $\sigma_{A \cup B} = \max\{\sigma_A, \sigma_B\}$.

For every finite set $S \subset \mathcal{S}$, let ϕ^S be the projection map from $\mathbb{R}^{\mathcal{S}}$ to $\mathbb{R}^S$. In Lemma 11 of DLR, the set $H = \{\lambda(\sigma_A - \sigma_B) \mid \lambda \geq 0 \text{ and } A, B \in X\}$ is shown to be a dense subset of the space of continuous functions over $\mathcal{S}$ under the sup norm topology. Since finite dimensional normed linear spaces are closed in the norm topology, an immediate consequence of this result is then:

Lemma 2. *For every finite $S \subseteq \mathcal{S}$ we have $\phi^S[H] = \mathbb{R}^S$.*

For any finite set $S \subset \mathcal{S}$, we will, abusing notation, identify the space $\mathbb{R}^S$ of functions from S to the reals with the euclidean space $\mathbb{R}^{|S|}$. Moreover we will identify each element of the canonical basis of $\mathbb{R}^{|S|}$ with the indicator function $\mathbb{1}_s$ for the state of the corresponding coordinate.

It will be useful to introduce a definition of FAEU (as referred to in Section 2.6) which is slightly different from the one given by Dekel Lipman Rustichini (2009). Say that a complete preference relation $\succcurlyeq$ over X has a **normalized** FAEU representation if there is a finite set $S \subset \mathcal{S}$ and a vector of weights $\mu \in \mathbb{R}^S$ such that $A \succcurlyeq B$ if and only if $W_\mu(A) \geqslant W_\mu(B)$, where

$$W_\mu(A) = \sum_{s \in S} \mu(s)\sigma_A(s).$$

We indicate a normalized FAEU representation with the pair (S, μ) or, in cases in which S is clear from the context, with the function $W_\mu : X \to \mathbb{R}$. Say that the vector $\mu \in \mathbb{R}^S$ induces the preference $\succcurlyeq$ on X if $A \succcurlyeq B$ if and only if $W_\mu(A) \geqslant W_\mu(B)$ for each $A, B \in X$. We call an element $s \in S$ such that $\mu(s) \neq 0$ a "state" of (S, μ). For any $s \in S$, let W_s stand for $W_{\mathbb{1}_s}$, and notice that $W_\mu = \sum \mu(s) W_s$. For a representation (S, μ), define the set of positive states as $P_\mu := \{s \in S \ |\mu(s) > 0\}$ and the set of negative states as $N_\mu := \{s \in S \ |\mu(s) < 0\}$. It is easy to show that for any two normalized FAEU representations (S, μ) and (S, μ') of $\succcurlyeq$, we must have $\mu = \lambda \mu'$, for some $\lambda > 0$. Also notice that by Lemma 1, each W_μ is continuous in the Hausdorff metric and affine in $+$, in the sense that $W_\mu(\alpha A + (1-\alpha)B) = \alpha W_\mu(A) + (1-\alpha) W_\mu(B)$ for all $\alpha \in (0,1)$.

We now present a partial characterization of preference structures $(\succcurlyeq, \succcurlyeq^*)$ that is an essential element of the proof of our main result:

Proposition 1. *If a preference structure $(\succcurlyeq, \succcurlyeq^*)$ over X satisfies Independence, Continuity and Finiteness, there is a sup-norm closed, convex cone $\mathcal{U}$ of functions from X to $\mathbb{R}$, continuous and affine in $+$, a finite set $S \subset \mathcal{S}$ and a vector $\mu^* \in \mathbb{R}^S$ such that*

1) *$A \succcurlyeq B$ if and only if $U(A) \geqslant U(B)$ for all $U \in \mathcal{U}$.*
2) *$U(\{\frac{1}{n}, ..., \frac{1}{n}\}) = 0$ for all $U \in \mathcal{U}$.*
3) *(S, W_{μ^*}) is a normalized FAEU representation of $\succcurlyeq^*$.*

Proof By Theorem 1 in Galaabaatar (2011), if $\succcurlyeq$ satisfies Independence and Continuity there is a sup-norm closed, convex cone $\mathcal{U}_0$ of functions from X to $\mathbb{R}$, that are continuous and affine in $+$, such that $A \succcurlyeq B$ if and only if $U(A) \geqslant U(B)$ for all $U \in \mathcal{U}_0$. Let $\mathcal{U} = \{U - U(\{\frac{1}{n}, ..., \frac{1}{n}\}) \mid U \in \mathcal{U}_0\}$. It is immediate that $\mathcal{U}$ is a closed convex cone satisfying items 1) and 2) of the proposition. Since $\succcurlyeq^*$ satisfies Independence, Continuity and Finiteness, by Theorem 6 in Dekel Lipman Rustichini (2009) it has a FAEU representation, from which a normalized FAEU representation (S, μ^*) can be easily derived. $\square$

To prove our main result we will need an additional claim which, under the Cautious Deferral Axiom, relates the FAEU representation of $\succcurlyeq^*$ to the functionals in $\mathcal{U}$. Before stating the claim, we introduce the following notation: indicate with $\mathbf{0}$ the element of $\mathbb{R}^X$ that is equal to 0 at every set $A \in X$. Notice that if a set of real valued functions over X satisfies item 2) of Proposition 1, the only constant function it contains is $\mathbf{0}$.

Claim 1. Let $\mathcal{U}$, S and μ^* characterize $(\succcurlyeq, \succcurlyeq^*)$ as per Proposition 1. Then, if $(\succcurlyeq, \succcurlyeq^*)$ satisfies Cautious Deferral, for every pair $A, B \in X$ such that 1) $W_{\mu^*}(A) > W_{\mu^*}(B)$ and 2) $W_s(B) > W_s(A)$ for all $s \in P_{\mu^*}$, we have

$$U(A) > U(B) \text{ for all } U \in \mathcal{U} \setminus \{\mathbf{0}\}$$

.

Proof: If A, B satisfy assumption 2) of the claim, $W_s(B) = \sigma_B(s) = \max\{\sigma_A(s), \sigma_B(s)\}$ for all $s \in P_{\mu^*}$. On the other hand, by Lemma 1, we have $\max\{\sigma_A(s), \sigma_B(s)\} = \sigma_{A \cup B}(s)$. Thus

$$W_{\mu^*}(B) = \sum_{s \in P_{\mu^*}} \mu^*(s) \sigma_B(s) + \sum_{s \in N_{\mu^*}} \mu^*(s) \sigma_B(s) \geqslant$$

$$\sum_{s\in P_{\mu^*}} \mu^*(s)\sigma_{A\cup B}(s) + \sum_{s\in N_{\mu^*}} \mu^*(s)\sigma_{A\cup B}(s) = W_{\mu^*}(A\cup B)$$

, since $\mu^*(s) < 0$ for all $s \in N_{\mu^*}$. As a consequence, for any two sets A, B satisfying 1) and 2), we must have $W_{\mu^*}(A) > W_{\mu^*}(A\cup B)$. This implies, by Cautious Deferral, that A and B are not incomparable. Since $\succcurlyeq^*$ is a completion of $\succcurlyeq$ we must then have $A \succ B$. Thus, by item 1) of Proposition 1, we obtain $U(A) \geq U(B)$ for all $U \in \mathcal{U}$ and $U'(A) > U'(B)$ for some $U' \in \mathcal{U}$.

Assume that for some $U \neq \mathbf{0}$, we have $U(A) = U(B)$. Since U is not constant, there are $A_1, B_1 \in X$ such that $U(A_1) < U(B_1)$. Since P_{μ^*} is finite and all the functions involved are continuous and affine w.r.t. $+$, there is an $\alpha \in (0,1)$ big enough such that the sets $A_0 = \alpha A + (1-\alpha)A_1$ and $B_0 = \alpha B + (1-\alpha)B_1$ satisfy

$$\begin{cases} W_s(B_0) > W_s(A_0) & \text{for all } s \in P_{\mu^*} \ , \\ W_{\mu^*}(A_0) > W_{\mu^*}(B_0), \\ U'(A_0) > U'(B_0), \\ U(A_0) < U(B_0). \end{cases}$$

Since the last two inequalities imply that $A_0 \bowtie B_0$ while the first $|P_{\mu^*}|+1$ inequalities imply that $W_{\mu^*}(A_0) > W_{\mu^*}(A_0 \cup B_0)$, this leads once again to a contradiction. □

Part 2: Proof of Theorem 1 If $\succcurlyeq$ is complete, the Cautious Deferral Axiom has no bite. Thus to prove Theorem 1 we essentially need to show that, given Axioms 1-4, if $\succcurlyeq$ is incomplete then $\succcurlyeq^*$ must be monotone. Let $\mathcal{U}$, S and μ^* characterize $(\succcurlyeq, \succcurlyeq^*)$ as per Proposition 1. We divide the proof in three cases:

Case 1: $\succ = \varnothing$. In this case for all $A \subseteq B$ we have either $A \sim B$ or $A \bowtie B$, so if $(\succcurlyeq, \succcurlyeq^*)$ satisfies Cautious Deferral, $\succcurlyeq^*$ must be monotone.

Case 2: $\succ \neq \varnothing$ and $P_{\mu^*} = \varnothing$. In this case $\succcurlyeq^*$ is always monotonically decreasing w.r.t. set inclusion, thus $W_{\mu^*}(A) > W_{\mu^*}(A\cup B)$ for all $A, B \in X$ such that $W_{\mu^*}(A) > W_{\mu^*}(B)$. By assumption, there are two incomparable sets A_1, B_1. By Cautious Deferral then it must be that $W_{\mu^*}(A_1) = W_{\mu^*}(B_1)$. But since $\succ$ is nonempty, we can find A_2, B_2 such that $W_{\mu^*}(A_2) > W_{\mu^*}(B_2)$.

It then follows, from affinity of W_{μ^*} and closed continuity of $\succcurlyeq$ that we can find $\alpha \in (0,1)$ big enough such that the sets $A_0 = \alpha A_1 + (1-\alpha)A_2$ and $B_0 = \alpha B_1 + (1-\alpha)B_2$ will satisfy $W_{\mu^*}(A_0) > W_{\mu^*}(B_0)$ and $A_0 \bowtie B_0$, leading to a contradiction.

Case 3: $\succ \neq \varnothing$ and $P_{\mu^*} \neq \varnothing$. If $N_{\mu^*} = \varnothing$, obviously $\succcurlyeq^*$ is monotone, so for the rest of the argument assume also that $N_{\mu^*} \neq \varnothing$. Since W_{μ^*} has both positive and negative states, we can find a vector $b \in \mathbb{R}^S$ such that $b_s > 0$ for all $s \in P_{\mu^*}$ while $\mu^* \cdot b < 0$. By Lemma 2, there exists $A, B \in X$ and a $\lambda > 0$ such that $b(s) = \lambda(\sigma_B(s) - \sigma_A(s))$ for every $s \in S$. Thus we can always find sets A, B satisfying the assumptions of Claim 1.

This implies that the Weak Pareto order induced by $\{-W_s\}_{s\in P_{\mu^*}} \cup \{W_{\mu^*}\}$ is nonempty. Since Claim 1 shows that for every $U \in \mathcal{U}\setminus\{\mathbf{0}\}$, the function U is increasing in such order, and all the functions considered are affine and equal to zero at $\{\frac{1}{n}, \ldots \frac{1}{n}\}$, we can apply Proposition 2 of De Meyer and Mongin (1995) to obtain that for each $U \in \mathcal{U} \setminus \{\mathbf{0}\}$ there are non-negative weights $\{\alpha_s^U\}_{s \in P_{\mu^*}}$ and β^U such that $U(A) = \beta^U W_{\mu^*}(A) - \sum_{s \in P_{\mu^*}} \alpha_s^U W_s(A)$.

Thus each $U \in \mathcal{U} \setminus \{\mathbf{0}\}$ is a function W_{μ^U} corresponding to the normalized FAEU representation (S, μ^U), where μ^U can be written as a non-negative linear combination of elements in $M_0 := \{\mu^*\} \cup \{-\mathbb{1}_s\}_{s \in P_{\mu^*}}$:

$$\mu^U = \beta^U \mu^* - \sum_{s \in P_{\mu^*}} \alpha_s^U \mathbb{1}_s. \tag{A.1}$$

From the properties of $\mathcal{U}$ it follows that $\mathcal{M} = \bigcup_{U \in \mathcal{U}} \mu^U$ is a closed convex cone. Moreover, since $\mathcal{M}$ is a cone in a finite dimensional space and $\succcurlyeq^*$ is a completion of $\succcurlyeq$, by a standard separation argument it can be shown that μ^* is in the relative interior of $\mathcal{M}$. Since $\succ \neq \varnothing$, there must be some μ in $\mathcal{M}$ not contained in the line spanned by μ^*.

If not, two cases may hold. The first one is that all $\mu \in \mathcal{M}$ induce the same order, namely $\succcurlyeq^*$, on X. But if $\succcurlyeq$ is incomplete, there are $A, B \in X$ such that $A \bowtie B$, which implies by item 1) of Proposition 1 that there are $\mu', \mu'' \in \mathcal{M}$ such that $W_{\mu'}(A) > W_{\mu'}(B)$ and $W_{\mu''}(A) < W_{\mu'}(B)$, a contradiction . The second is that they induce $\succcurlyeq^*$ and its dual $\preccurlyeq^*$, in which case there can be no $A, B \in X$ such that $A \succ B$. So $\mathcal{M}$ has at least dimension 2. Thus we can find two elements μ^1 and μ^2 of $\mathcal{M}$ that are linearly independent and such that μ^* is contained in the relative interior of the convex cone they span, which we denote with $co(\mu^1, \mu^2)$.

Let M_i be the subset of M_0 whose elements have strictly positive weight in the representation of form (A.1) of vector μ^i, for $i = 1, 2$. Then by Theorem 6.9 in Rockafellar (1970), the relative interior of $co(\mu^1, \mu^2)$ is contained in the relative interior of $co(M_1 \cup M_2)$. In particular, μ^* is in the relative interior of $co(M_1 \cup M_2)$. Since $co(\mu^1, \mu^2)$ is a two dimensional object, it cannot be that $co(M_1 \cup M_2) = \{\lambda \mu^* \,|\, \lambda \in \mathbb{R} \text{ and } \lambda > 0\}$. This implies that $\mu^* \in co(M_1 \cup M_2) = co(\{M_1 \cup M_2\} \setminus \{\mu^*\})$, so there are non negative weights $\{\alpha_s^{\mu^*}\}_{s \in P_{\mu^*}}$ such that $W_{\mu^*} = -\sum_{s \in P_{\mu^*}} \alpha_s^{\mu^*} W_s$. But then W_{μ^*} has no positive states, contradicting our assumption that $P_{\mu^*} \neq \varnothing$. ■

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