Cautious Deferral, Indecisiveness and Preference for Flexibility

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For those who want to leave the room now ...

... here is a summary of what we find:

"*Indecision is the key to flexibility*." Proverb

For those who want to stay, please hold on to this message.

Flexibility versus Commitment (1)

- The concepts of preference for flexibility and commitment have been widely studied in decision theory.
- How? By defining a preference relation $\succeq$ over a set of opportunity sets or "menus":

Preference for flexibility: If $B \succeq A$ for all A, B s.t. $A \subseteq B$.

  - Kreps (1979), Dekel, Lipman & Rustichini (2001)

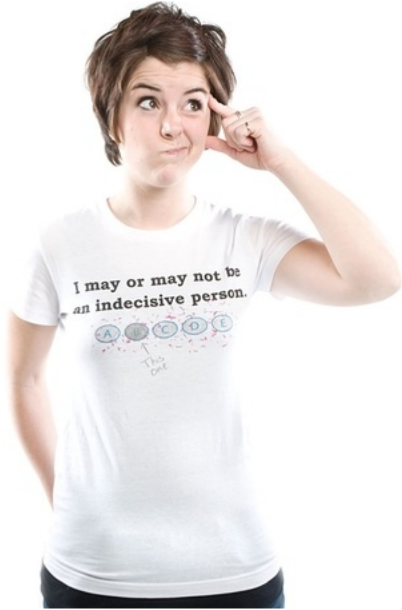
Preference for commitment: If $A \succeq B$ for all A, B s.t. $A \subseteq B$.

  - Gul and Pesendorfer (2001), Dekel, Lipman & Rustichini (2001, 2009), Sarver (2008)

Flexibility versus Commitment (2)

- Justification for preference for flexibility: the presence of **unforeseen contingencies**
  - Kreps (1979) and DLR (2001)
- Justifications for commitment: **temptation, regret**
  - Gul and Pesendorfer (2001), Sarver (2008)
- However, these are *interpretations* from the representation.
- Are there specific circumstances where we should expect one property or the other to apply?

Flexibility Under Conflict (1)



Flexibility Under Conflict (2)

In words, the rule:

"*Whenever in doubt, don't commit now; just leave options open*."

... seems rather intuitive and well supported by empirical evidence.

Beyond anecdotal evidence, several studies show that whenever conflicted between two options, people indeed tend to differ choice.

Flexibility Under Conflict (3)

Tversky and Shafir (1992) - Study 2

- Subjects asked to make an hypothetical choice between 3 options:
  1. buy a popular $99 SONY player
  2. buy a top-of-the-line $159 AIWA player
  3. wait until you learn more about the various models
- They find:
$$\%_{\{1\}} = \%_{\{2\}} = 27 < \%_{\{3\}} = 46$$
- On the other hand, when 2 is replaced by an inferior option:
$$\%_{\{2\}} = 0 < \%_{\{3\}} = 24 < \%_{\{1\}} = 76$$

Flexibility Under Conflict (4)

Deferral may be valued as a way to:

- acquire more information
- obtain additional time for contemplation

Tykocinski & Ruffle (2003)

- Students made a choice between the following options:
  1. register for a course of either uncertain/certain quality
  2. not register for the course
  3. wait until tomorrow
- What they find:
  - In the uncertain scenario, % wait = 48.
  - In the certain scenario, % wait = 17.

What we do:

We introduce this intuitive rule in the standard framework of menu choice:

1. We consider **two preferences relations** $\succeq$ and $\succeq^*$ where:
   - $\succeq$ represents the psychological tastes of the DM; possibly incomplete.
   - $\succeq^*$ is a completion of $\succeq$ which captures the DM's behavior.
2. We connect $\succeq$ and $\succeq^*$ through a new axiom called **Cautious Deferral** which formalizes our intuitive rule.
   - Technically, we restrict flexibility to apply to only part of the domain: whenever the agent cannot compare alternatives.

What we ask:

1. What are the restrictions imposed by the Cautious Deferral Axiom?

2. Is there a trade-off between commitment and flexibility?

What we find:

1. Provided $\succeq$ is not complete, $\succeq^*$ **must be monotone on its *entire* domain**.
    - Cautious Deferral alone forces $\succeq^*$ to exhibit preference for flexibility.
    - In his sense, *"Indecision is the key to flexibility*."
2. There is a fundamental **tension between non-monotonic preferences, indecisiveness and Cautious Deferral**.
    - Consider a DM with incomplete preferences who satisfies Cautious Deferral.
    - Then he cannot exert both commitment and flexibility.

The model: basic setup

- We adopt the standard framework of menu preferences of DLR (2001).
- Let $\triangle$ be the set of all probability distributions on a prize space Z where $\mid Z \mid = n < \infty$ and $p, q, r, ... \in \triangle$.
- Let X be the set of all non-empty closed subsets of $\triangle$ endowed with the Hausdorff metric. $A, B, C, ... \in X$ are *menus.*
- Define the following mixture operation $+$ on X:

$$\lambda A + (1 - \lambda)B := \{\lambda p + (1 - \lambda)q \mid p \in A \text{ and } q \in B\}$$

where $\lambda \in [0, 1]$.

The model: preference structures

Our primitive is a *preference structure* $(\succeq, \succeq^*)$ on X where:

1. $\succeq$ is a preference relation on X (reflexive and transitive)
2. $\succeq^*$ is a proper completion of $\succeq$ i.e.

$$A \succeq B \text{ implies } A \succeq^* B \text{ and } A \succ B \text{ implies } A \succ^* B$$

Interpretation:

$\Rightarrow$ $\succeq$ represents the psychological tastes of the DM.

$\Rightarrow$ $\succeq^*$ represents the revealed preferences of the DM.

Basic Axioms

Axiom 1: (Independence): For every $A, B, C \in X$ and $\lambda \in (0,1)$,

$$A \succeq B \text{ if and only if } \lambda A + (1-\lambda)C \succeq \lambda B + (1-\lambda)C$$

and similarly for $\succeq^*$.

Axiom 2 (Continuity): Both $\succeq$ and $\succeq^*$ are closed in $X \times X$.

Definition ($\succeq^*$- criticality): $A_0 \subseteq conv(A)$ is $\succeq^*$-critical for A if

$$B \sim^* A \text{ for all } B \in X \text{ s.t. } A_0 \subseteq conv(B) \subseteq conv(A).$$

Axiom 3 (Finiteness): Every menu in X has a finite $\succeq^*$-critical subset.

The Cautious Deferral Axiom

Denote by $\bowtie$ the non-comparability part of $\succeq$.

Axiom 4 (Cautious Deferral): For every A, $B \in X$,

$$A \bowtie B \text{ implies } A \cup B \succeq^* A$$

$\Rightarrow$ "*Whenever in doubt, don't commit; just leave options open*."

$\Rightarrow$ One can interpret preference for flexibility as choice deferral.

$\Rightarrow$ Descriptive rather than normative property.

Main result and interpretation

Theorem 1: Let $(\succeq, \succeq^*)$ be a preference structure on X which satisfies Axioms $1-4$. Then, either $\succeq$ is complete or $\succeq^*$ satisfies preference for flexibility.

$\Rightarrow$ If $\bowtie = \{(A, B) : A \neq B\}$, $\succeq^*$ must be monotone.

$\Rightarrow$ If $\bowtie = \varnothing$, Cautious Deferral has no bite.

$\Rightarrow$ If $\exists$ A, B s.t. $A \bowtie B$, $\succeq^*$ must be monotone on its *entire* domain.

1. Provides a psychological foundation to preference for flexibility.
2. Impossibility result: so long as $\bowtie \neq \varnothing$, Cautious Deferral completely precludes commitment.

The main result: interpretation (continued)

- Theorem 1 however doesn't say that the core relation $\succeq$ has to be monotonic.
- In fact, it need not be. As a counterexample, define

$$U(A) := \lambda \max_{p \in A} u(p) - \max_{p \in A} v(p)$$

$$V(A) := \lambda \max_{p \in A} v(p) - \max_{p \in A} u(p)$$

where u and v are distinct, continuous and affine on $\triangle$ and $\lambda > 1$.

- Define $\succeq$ as $A \succeq B$ iff $U(A) \geq U(B)$ and $V(A) \geq V(B)$.
- Let $\succeq^*$ be represented by $U + V$.
- $\succeq$ is incomplete and $\succeq^*$ is a monotone completion satisfying A1 - A4 but $\succeq$ is not monotone.

Corollary of the main theorem

We can ensure the monotonicity of $\succeq$ by making the following changes to the axioms:

Axiom 3' ($\succeq$- Finiteness): Every menu has a finite $\succeq$-critical subset.

Axiom 4' ($\succeq$- Cautious Deferral): For every A, $B \in X$ s.t. $A \not\subseteq B$ and $B \not\subseteq A$,

$$A \bowtie B \text{ implies } A \cup B \succeq A$$

Theorem 2 (Corollary): Let $\succeq$ be a preference relation on X which satisfies Axioms 1, 2, 3' and 4'. Then $\succeq$ is either complete or it exhibits a preference for flexibility.

Self-Control versus Cautious Deferral

Theorem 1 can be interpreted as a negative result: one must forget about commitment if Cautious Deferral has some bite.

- To illustrate this point, consider the model of temptation of Gul and Pesendorfer (2001). Their key axiom is:

  Set-Betweenness: $A \succeq^* B$ implies $A \succeq^* A \cup B \succeq^* B$.

- In our framework, suppose we wanted to impose:
  1. $A \succeq B$ implies $A \succeq A \cup B$ (and hence $A \succeq^* A \cup B$)
  2. $A \bowtie B$ implies $A \cup B \succeq^* A$

- Then we must have a standard DM. Indeed, take any A, B s.t. $A \succeq^* B$. We have:
  1. By SB, $A \succeq^* A \cup B$.
  2. By Theorem 1, $A \cup B \succeq^* A$.
  3. Thus $A \succeq^* B$ implies $A \sim^* A \cup B$.

Sketch of the proof (1)

Definition: Finite additive Expected Utility (FAEU) - DLR (2009)

A complete binary relation on X has a FAEU if it can be represented by a map $W : X \to \mathbb{R}$ defined by

$$W(A) = \sum_{i \in P} U_i(A) - \sum_{j \in N} V_j(A)$$

where $U_i(A) = \max_{p \in A} u_i(p)$ and $V_j(A) = \max_{p \in A} v_j(p)$

u_i and v_j are EU; P and N are finite index sets.

Remarks:

- W is continuous and affine.
- There are $\mid P \mid + \mid N \mid$ subjective states of the world where $i \in P$ is a positive state and $j \in N$ a negative state.
- If $W(A \cup B) \geq W(B)$ then $U_i(A) \geq U_i(B)$ for some $i \in P$.

Sketch of the proof (2)

Step 1: If $(\succeq, \succeq^*)$ satisfies A1 - A3, then:

1. $\succeq = \cap_{k \in \mathcal{K}} \succeq_k$ where each $\succeq_k$ is complete and represented by some continuous and affine W_k.
2. $\succeq^*$ has a FAEU representation $W^* = \sum_{i \in P^*} U_i^* - \sum_{j \in N^*} V_j^*$

Let's focus on the non-trivial cases where $\succ \neq \varnothing$ and $P^* \neq \varnothing$.

1. If $\succ = \varnothing$, $\succeq^*$ is trivially monotone.
2. If $P^* = \varnothing$, Cautious Deferral must be violated.

Sketch of the proof (3)

Step 2: If A4 applies, then $W_k = \beta_k W^* - \sum_{i \in I} \alpha_k^i U_i^*$ where β_k, $\alpha_k^i \geq 0$, for all $k \in \mathcal{K}$.

1. If $U_i^*(B) > U_i^*(A)$ for all $i \in P$ then $W^*(B) \geq W^*(A \cup B)$.
2. In addition, if $W^*(A) > W^*(B)$ then $W^*(A) > W^*(A \cup B)$.
3. By CD, we cannot have $A \bowtie B$. So, we must have $A \succ B$.
4. With A1 - A2, one can show that $W_k(A) > W_k(B)$ for all $k \in \mathcal{K}$.
5. Thus each W_k is increasing in the Weak Pareto order induced by $\{W^*, -U_1^*, ..., -U_P^*\}$.

   Step 2 follows from an aggregation theorem à la Harsanyi.

Sketch of the proof (4)

Step 3: Contradiction

1. Using a separation argument, we can show that W^* must lie in the relative interior of the cone spanned by the W_k's.
2. On the other hand, each W_k must lie in the cone spanned by W^* and the $-U_i$'s.
3. Only two cases can occur:

- $W_k = W^*$ for all $k \in \mathcal{K}$.
- Each W_k must lie in the cone spanned only by the $-U_i's$.

In the first case, $\succeq$ must be complete.

In the second, $P^* = \varnothing$. $\Rightarrow\Leftarrow$

What happened here?

- Well... we wish we knew!
- Clearly, closed continuity and Independence are essential in the proof.
- In particular, Independence gives us affinity.
- We also conjecture that with open continuity, the result wouldn't go through.
- Combined together, they give strength to Cautious Deferral.

What's next?

- Find someone smart to explain us the proof.
- Work out a couple of examples.
- Any suggestions? Otherwise...

The End

... let me finish with a last quote: